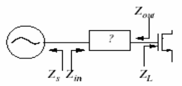
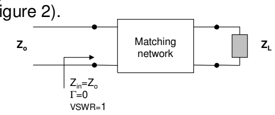
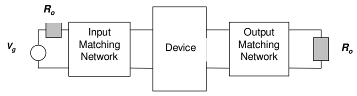
Impedance Matching and Tuning

- **Matching networks are used to match the impedance of one system to another**
- **Match is important for several reasons:**
  - **– Provides for maximum power transfer (e.g. carrying power from transmitter to an antenna).**
  - **– Improves the signal to noise ratio (e.g. when carrying input signals from an antenna to a receiver).**
  - **– Reduces amplitude and phase errors in power distribution networks (e.g. when designing a distribution network for an antenna array).**
  - **– Reduces the VSWR, increasing the maximum power-transfer ability of high-power transmission systems**
- E.g. if the input to a transistor at a particular frequency is taken to be $Z_L = 50 - j60$ and source applied to the input of the transistor has an output impedance of $Z_s = 50$, we need to find a matching network such that $Z_{in} = Z_s^*$, or equivalently $Z_{out} = Z_L^*$



•When a load connected to a line whose characteristic impedance differs from the load, there will be reflections resulting in a reduction in the power delivered to the load.
•To maximize the power delivered to the load, or equivalently reduce the reflection, a lossless impedance network should be inserted between the load and the line. This is called one-port impedance matching as depicted in Figure 1.
•For devices such as filters, amplifiers etc, both input matching and output matching are required to match the input and the output of the device so that maximum power should be delivered, correspondently low reflection is obtained (Figure 2).





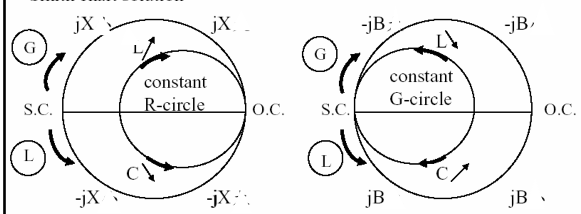
What is a matching network?

- Lumped and/or distributed elements
- Lumped elements
  - – Consists of discrete components (resistors, inductors, capacitors, and possibly active devices)
  - – Usually more compact
  - – At high frequencies, parasitics reduce the effectiveness of the components
  - – Limited by practicality of components (e.g. can't have 1 mH inductor on monolithic circuit)
- Distributed
  - – Uses series transmission lines and stubs
  - – Take up much more space (which means larger cost for MMIC)
  - – Reactive parasitics associated with these usually much lower
  - – Ohmic losses may be significant due to their large size

Both of these matching networks can be easily designed using - SMITH CHARTS!

Matching With Lumped Elements (L Networks)

- Smith chart solution



Z-plane	Y-plane
CW → increase series L (reduce series C)	CW → increase shunt C (reduce shunt L)
CCW → increase series C (reduce series L)	CCW → increase shunt L (reduce shunt C)

Remember impedance-admittance terminology

The Smith chart can be used for line admittances, by shifting the space reference to the admittance location. After that, one can move on the chart just reading the numerical values as representing admittances.

Let's review the impedance-admittance terminology:

Impedance = Resistance + j Reactance

$$Z = R + jX$$

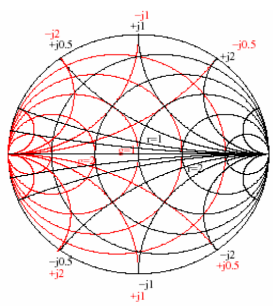
Admittance = Conductance + j Susceptance

$$Y = G + jB$$

On the impedance chart, the correct reflection coefficient is always represented by the vector corresponding to the normalized impedance. Charts specifically prepared for admittances are modified to give the correct reflection coefficient in correspondence of admittance.

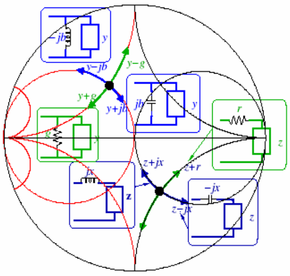
Impedance/admittance Smith chart

- Admittance chart is rotated 180° relative to the impedance chart
- Now black lines indicate constant impedance, and red lines constant admittance



Motion on Smith Chart

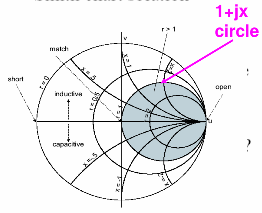
- Simplest way to visualize the effects of matching uses the Smith chart
- Addition of various lumped and distributed elements can be seen as "movement" around the Smith chart
- For a particular normalized load with impedance z, or admittance $y = 1/z$, the effect of adding various elements is shown to the right
- Series inductor: positive reactance, move clockwise on resistance circle
- Series capacitor: negative reactance, move counter-clockwise on resistance circle
- Parallel inductor: negative admittance, move counter-clockwise on conductance circle
- Parallel capacitor: positive reactance, move clockwise on conductance circle



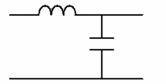
There are two possible configurations for this network

1. ZL inside 1+jx circle, two possible solutions

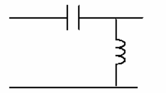
Smith chart solution



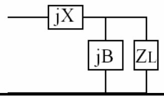
A:



B:



analytical solution



$$Z_o = jX + \cfrac{1}{jB + \cfrac{1}{R_L + jX_L}}$$

$$\Rightarrow B > 0 \to C, B < 0 \to L$$

$$X > 0 \to L, X < 0 \to C$$

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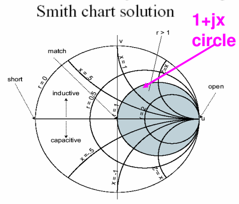
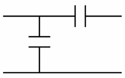
2. ZL outside 1+jx circle, two possible solutions

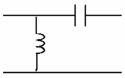
Smith chart solution



A



B



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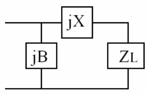
analytical solution



$$\frac{1}{Z_o} = jB + \frac{1}{R_L + j(X + X_L)}$$

$$\Rightarrow B > 0 \to C, B < 0 \to L$$

$$X > 0 \to L, X < 0 \to C$$

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1. Normalize: $z_L = Z_l/Z_o$
2. Decision:

z_L inside $r=1$ circle (Type "A")

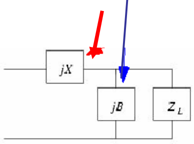
a. Move along a constant conductance circle until it hits the **g=1** circle (i.e. we are adding a susceptance *jb* parallel to the load)

b. **Convert back to impedance (i.e. adding a reactance *jx* in series**

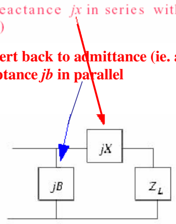
z_L outside $r=1$ circle (Type "B")

a. Move along a constant resistance circle until it hits the $g=1$ circle (i.e. we are adding an reactance *jx* in series with the load)

b. **Convert back to admittance (ie. adding a susceptance *jb* in parallel**



A



B

Second Method

1. Normalize: $z_L = Z_l/Z_o$
2. Decision:

z_L inside $r=1$ circle (Type "A")

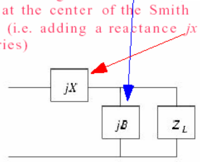
a. Move along a constant conductance circle until it hits the $r=1$ circle (i.e. we are adding a susceptance *jb* parallel to the load)

b. Move along the $r=1$ circle until at the center of the Smith chart (i.e. adding a reactance *jx* in series)

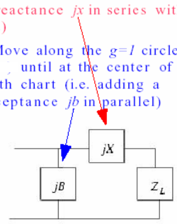
z_L outside $r=1$ circle (Type "B")

a. Move along a constant resistance circle until it hits the $g=1$ circle (i.e. we are adding an reactance *jx* in series with the load)

b. Move along the $g=1$ circle until at the center of the Smith chart (i.e. adding a susceptance *jb* in parallel)



A



B

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3. "Un-normalize" the component impedances/admittances:

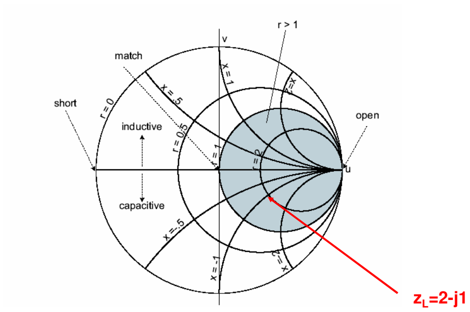
   $jX = Z_o jx, \; jB = jb/Z_o$

4. Calculate the value of inductors/capacitors required to implement the jX and jB at the specified frequency.
5. Any load that does not sit on the $r=1$ circle or the $g=1$ circle will have two possible matching solutions, as we can always intersect the $r=1$ circle or the $g=1$ circle at two places.

Example

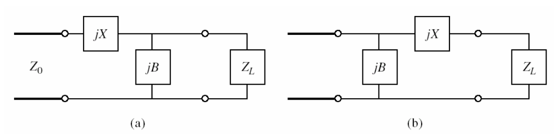
- Design an L-section matching network to match a series RC load with an impedance Z_L=*200-j100Ω*, to a *100Ω* line, at a frequency of *500 MHz*.

Normalize $z_L=Z_L/Z_0$, z_L=2-j1

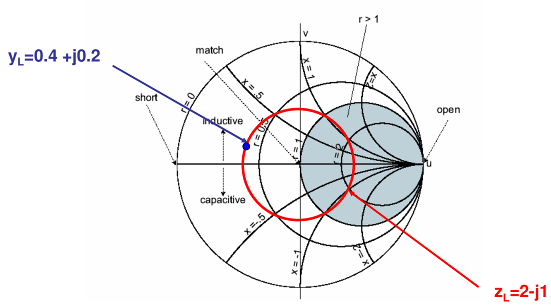


Decision: z_L inside r=1 circle (Type "A")

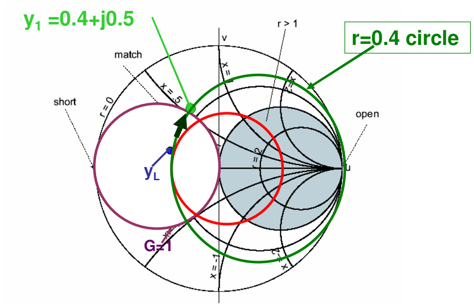
This point inside 1+jx circle so we will use the matching circuit of fig(a)



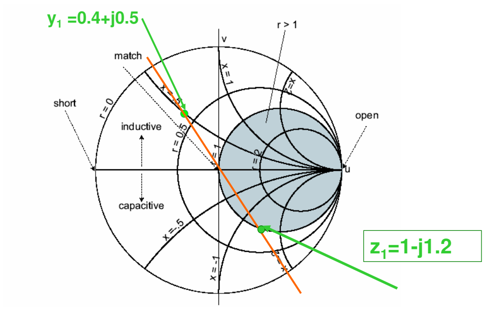
Plot SWR circle and find y_L



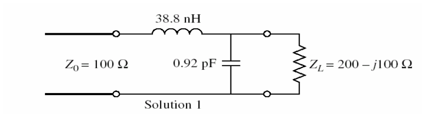
Move along a constant conductance circle



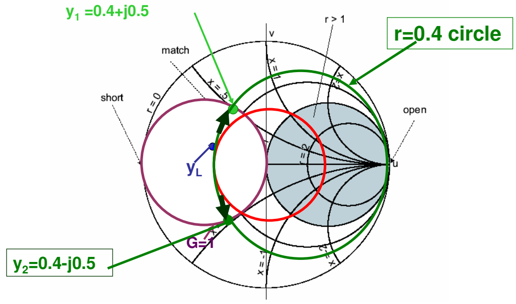
- y_1-y_L=(0.4+j0.5)-(0.4+j0.2)=j0.3=jb
- Adding a susceptance of *jb=j0.3* will move us along a constant conductance circle to y_1=0.4+j0.5
- After we add the shunt susceptance and convert back to impedance, we want to be on the 1+jx circle.
- So that we can add a series reactance to cancel the jx and match the load.



- z_1=1-j1.2, indicating that a series reactance *x=j1.2* will bring us to z=1.($Z=Z_0$)
- This matching circuit consist of shunt capacitor (*b>0*) and a series inductor (*x>0*).
- shunt capacitor: $jB=j\omega C$ and $B=b/Z_0$
  $C=b/2\pi fZ_0=0.3/2\pi 500{\times}10^6{\times}100=0.92$ pF
- Series inductor: $jX= j\omega L$ and $X=x.Z_0$
  $L=xZ_0/\ 2\pi f=38.8$ nH

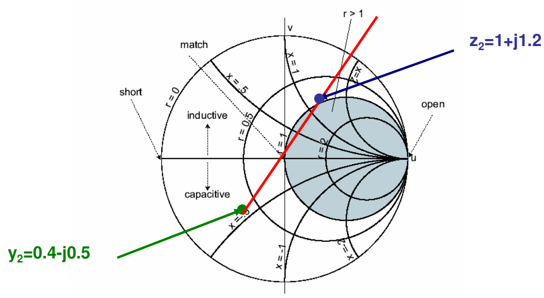


Second solution to this matching problem: We will move to point on the lower half of the shifted 1+jx circle



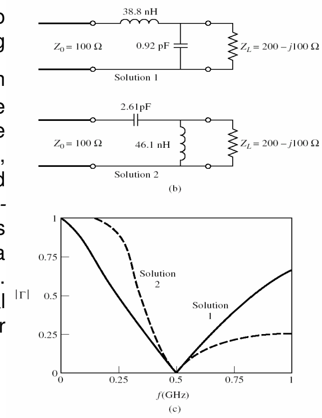
- y_2-y_L=(0.4-j0.5)-(0.4+j0.2)=-j0.7=jb
- Adding a susceptance of *jb=-j0.7* will move us along a constant conductance circle to y_1=0.4-j0.5
- After we add the shunt susceptance and convert back to impedance, we want to be on the 1+jx circle.
- So that we can add a series reactance to cancel the jx and match the load.

Second solution to this matching problem: continued



- z_2=1+j1.2, indicating that a series reactance *x=-j1.2* will bring us to z=1.($Z=Z_0$)
- This matching circuit consist of shunt inductor (*b=-0.7<0*) and a series capacitor (*x=-1.2<0*).
- shunt inductor: *jB=-j/ωL* and $B=b/Z_0$
  L=-Z_0/2 π fb=-100/ /2π500x10^6x(-0.7)=46.1 nH
- Series capacitor: *jX= -j1/ωC* and $X=x.Z_0$
  C=-1/ 2πfxZ_0=2.61 pF

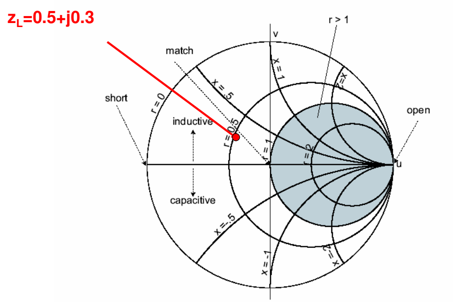
Figure shows the two possible L-section matching circuits and the reflection coefficient magnitude versus frequency for these two matching networks, assuming that the load impedance of Z_L*=200-j100Ω* at 500 MHz consists of a *200Ω* resistor and a *3.18 pF* capacitor in series. There is no substantial difference in bandwidth for these two solutions.



Example

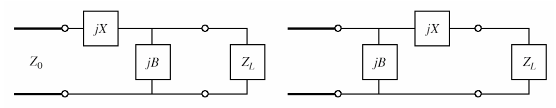
- Design lossless L-section matching network for the following normalized load impedance: z_L=0.5+j0.3

$z_L=Z_L/Z_0$ =0.5+j0.3



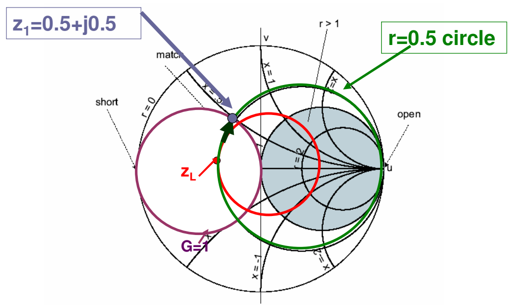
Decision: z_L outside r=1 circle (Type "B")

This point outside 1+jx circle so we will use the matching circuit of fig(b)

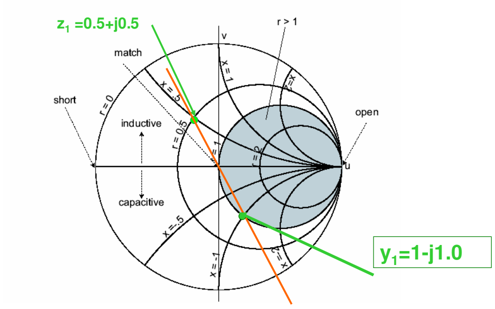


(a) (b)

Move along a constant reactance circle until it hits g=1 circle



- z_1-z_L =(0.5+j0.5)-(0.5+j0.3)=j0.2=jx
- Adding a reactance of *jx=j0.2* will move us along a constant reactance circle to $z_1=0.5+j0.5$
- After we add the series reactance and convert back to admitance,
- So that we can add a shunt suceptance to cancel the jb and match the load.



- $y_1=1-j1.0$, indicating that a shunt suceptance *b=j1.0* will bring us to $y=1 (y=1/Z_0)$.
- This matching circuit consist of shunt capacitor *(b>0)* and a series inductor *(x>0)*.

Second solution to this matching problem:

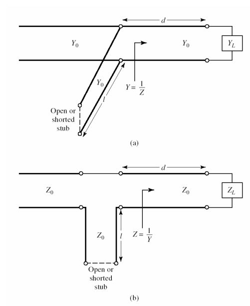
- x_2=-0.8
- b_2=-1

Matching Networks Using TL

- •Single-Stub Tuning
- •Double-Stub Tuning

Single Stub Tuning

- We next consider a matching technique that uses a single open-circuited or short-circuited length of transmission line (a "stub").



- Here, it is important whether we should use
  - short circuit or
  - open circuit stub in the design.
- Due to excessive radiation loss, short circuit stub is preferred if coaxial cable is used.
- On the other hand, for printed circuit board (PCB) design, open-circuit stubs are preferred since they do not require via that is necessary to obtain the ground connection for a short circuit stub.

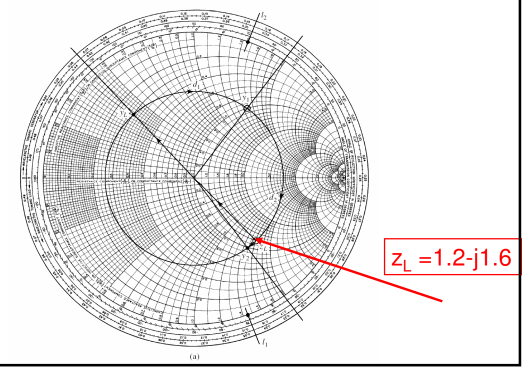
- Single stub impedance matching is easy to do with the Smith Chart
- Simply find the intersection of the SWR circle with the *r = 1* circle
- The match is at the center of the circle. Take a reactance in series or shunt to move you there!

Single-Stub Shunt Tuning

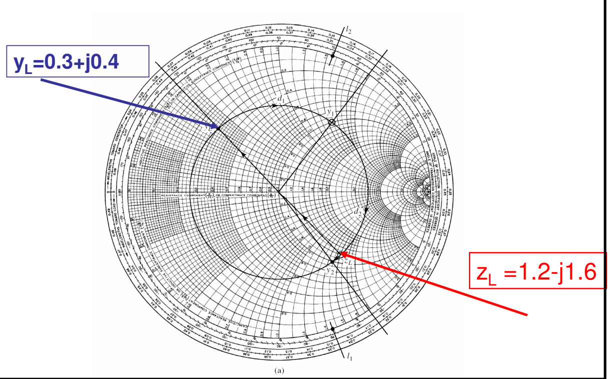
Example:

For a load impedance $Z_L=60-j80\Omega$, design two single-stub (short circuit) shunt tuning networks to match this load to a 50Ω line. Assuming that the load is matched at 2 GHz, and that the load consist of a resistor and capacitor in series, Plot the reflection coefficient magnitude from 1 GHz to 3 GHz for each solution

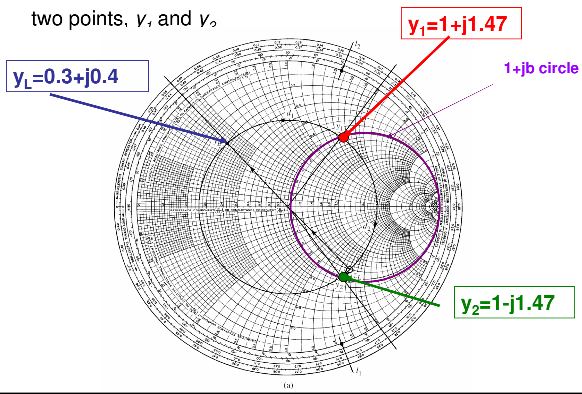
Locate the normalized load impedance ($z_L=1.2-j1.6$) and Plot SWR circle



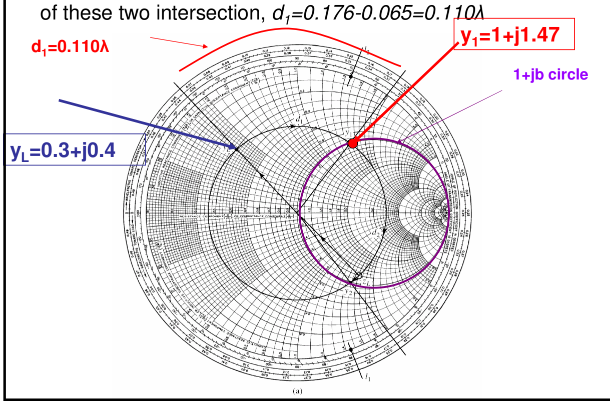
Find y_L



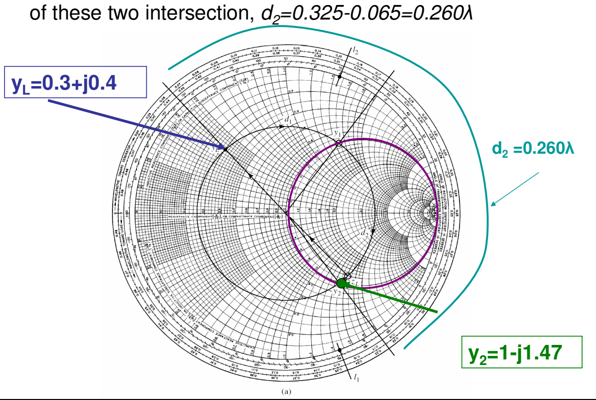
For the remaining steps we consider the Smith chart as an admittance chart. SWR circle intersects the *1+jb* circle at two points, y_1 and y_2



The distance d, form the load to the stub, is given by either of these two intersection, $d_1=0.176-0.065=0.110\lambda$



The distance d, form the load to the stub, is given by either of these two intersection, $d_2=0.325-0.065=0.260\lambda$

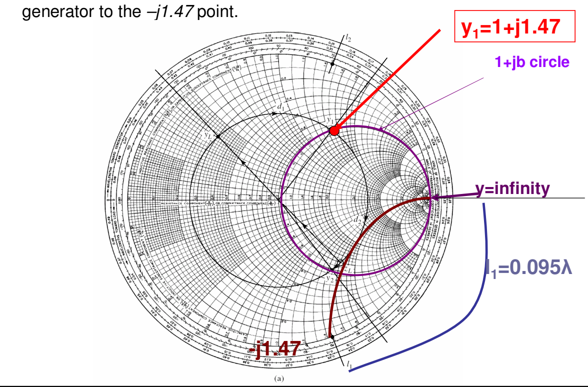


- At the two intersection points, the normalized admittances are

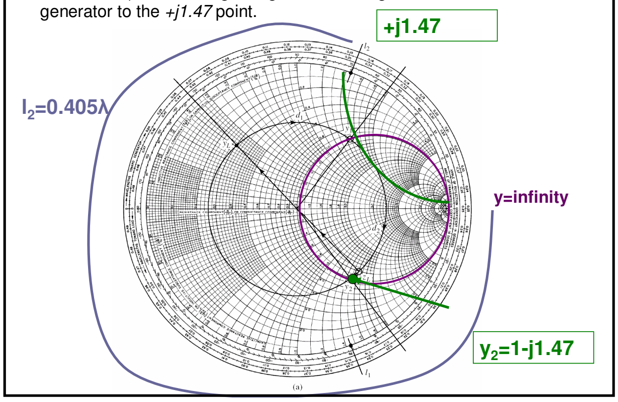
$y_1=1+j1.47$ (a stub with a susceptance –j1.47)

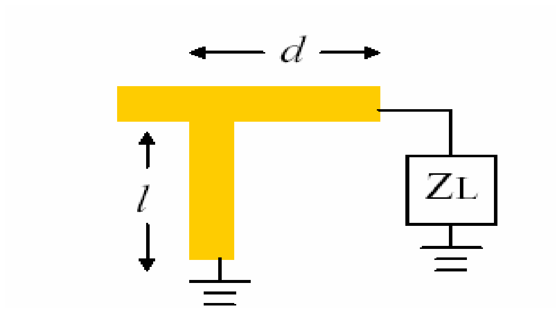
$y_2=1-j1.47$ (a stub with a susceptance +j1.47)

The length of the short circuited stub *(l_1)* : by starting at *y=infinity* (the short circuit) and moving along the outer edge of the chart toward the generator to the *–j1.47* point.

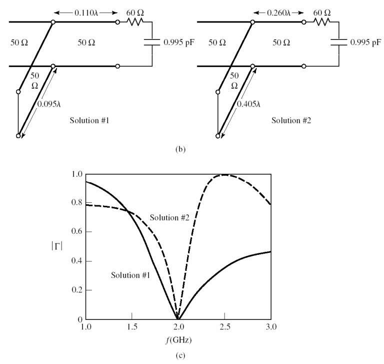


The length of the short circuited stub *(l_2)* : by starting at *y=infinity* (the short circuit) and moving along the outer edge of the chart toward the generator to the *+j1.47* point.





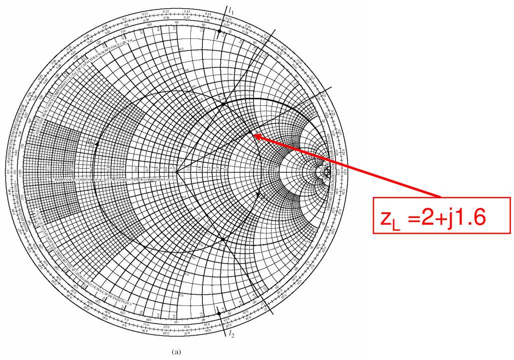
- The series RL load impedance is Z_L=*60-j80Ω* at 2 GHz, so *R=60 Ω* and *C=0.995 pF.*
- The two tuning circuits are shown.



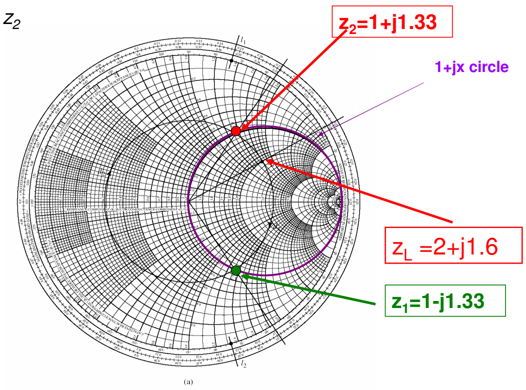
Single-Stub Series Tuning

Example: Match a load impedance of Z_L*=100+j80* to a *50Ω* line using a single series open-circuit stub. Assuming that the load is matched at *2 GHz*, and that the load consist of a resistor an inductor in series, plot the reflection coefficient magnitude from *1 GHz* to *3 GHz*.

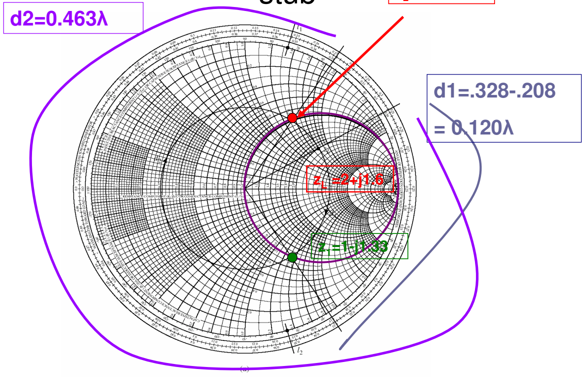
Locate the normalized load impedance (z_L*=2.0+j1.6*) and Plot SWR circle



For the series-stub design the chart as an impedance chart. SWR circle intersects the *1+jx* circle at two points, z_1 and z_2



The distance *d*, form the load to the stub

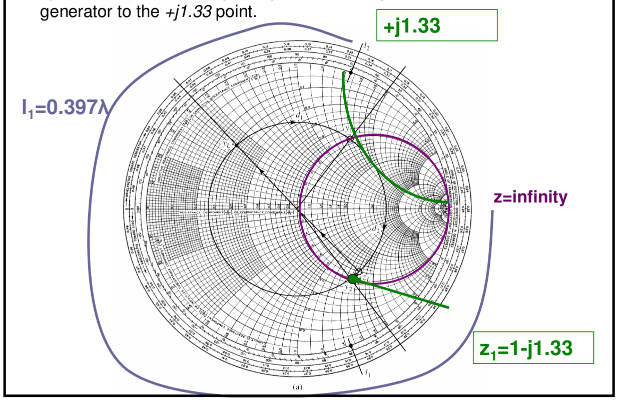


- At the two intersection points, the normalized impedances are

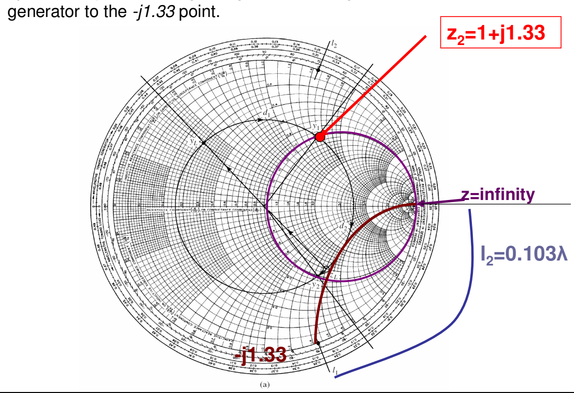
z_1=1-j1.33 (a stub with a reactance of +j1.33)

z_2=1+j1.33 (a stub with a reactance of -j1.33)

The length of an open circuited stub *(l_1)* : by starting at *z=infinity* (the open circuit) and moving along the outer edge of the chart toward the generator to the *+j1.33* point.

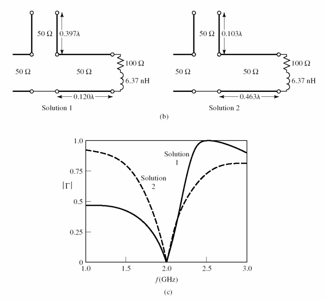


The length of an open circuited stub *(l_2)* : by starting at *z=infinity* (the open circuit) and moving along the outer edge of the chart toward the generator to the *-j1.33* point.



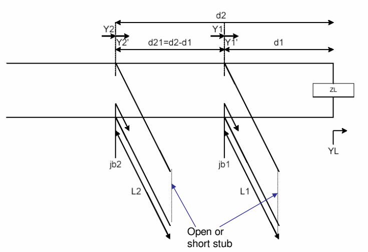
•The series RL load impedance is *ZL=100+j80Ω* at 2 GHz, so *R=100 Ω* and *L=6.37 nH.*

•The two tuning circuits are shown.



Double-Stub Tuning

- A single stub must be repositioned to match for different loads. Sometimes this is inconvenient. The double stub method fixes the location of the stubs and varies the lengths. Usually the stubs are separated by standard distances: λ/4, λ /8, or 3 λ /4.
- The stubs are shunt stubs, which are usually easier to implement in practice than are series stubs.



The circuit is to be matched at the beginning of the matching network. This implies that $Y_2 = 1+0j = Y'_2+jb_2$ ➔ $Y'_2 = Y_2 - jb_2 = 1-jb_2$ The admittance just after the second stub must fall on the unit circle of conductance (g = 1)

Y_1 is Y'_2 delayed by the distance d_{21}: WTL (CCW) a distance d_{21} this gives the auxiliary circle (g =1 circle rotated by the distance d_{21})

$Y_1 = Y'_1 + jb_1$

Y'_1 is the load admittance rotated by the distance from the load.

Goal is to get to origin of Smith Chart where *r =1*, *g =1*, and *Γ= 0*

Method:

1 Normalize load and plot (A)
2 Draw SWR circle
3 Reflect through origin to get admittance (B)
4 Rotate WTG to position of first stub
5 Draw Auxiliary Circle by rotating g=1 circle by (CCW)
6 Shift to Aux. Circle by adding stub1 (C&C')
7 Rotate d21 to second position of second stub (new SWR circle) (D&D')
8 Add stub to cancel imaginary part (to origin)

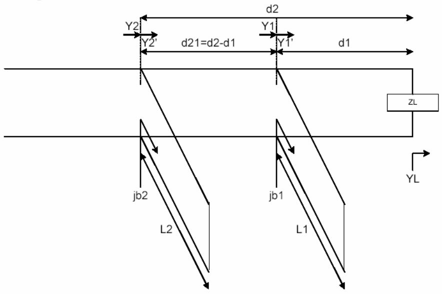
EXAMPLE:

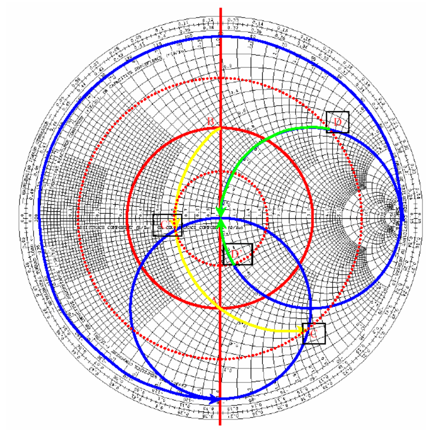
$Z_0 = 50$

$Z_L = 30 - 40j$

$d_1 = 0$

$d_2 = .375\lambda$





1 $\bar{z}_L = \frac{Z_L}{Z_0} = \frac{30-40j}{50} = 0.6 - 0.8j$ (A)

2 SWR Circle

3 $\bar{y}_L = 0.6 + 0.8j$ (B)

4 Rotate WTG to position of first stub ($d_1 = 0$) no rotation

5 Draw Aux. Circle , rotate g = 1 circle $d_{21} = .375\lambda$

6 $y_1 = y_L + jb_1$ ➔ $y_1 - y_L = jb_1$ (C) = 0.6-1.9j, (C') = 0.6-.09j

C: 0.6-1.9j-(0.6+0.8j) = -2.7j ➔ .306λ-.25λ=.056λ

C':0.6-0.09j-(0.6+0.8j) = -0.89j ➔ .384λ-.25λ = .134λ

7 D: 1+2.5j D': 1-0.53j

8 $b_2j = -2.5j$ ➔ .31λ-0.25λ=.06λ

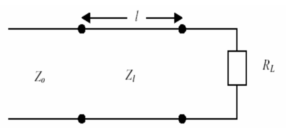
$b_2'j = 0.53j$ ➔ .08λ+.25λ=.33λ

Try these problems and see if you get these answers:

$Z_0 = 50$	$Z_0 = 50$
$Z_L = 60 - 80j$	$Z_L = 80 + 60j$
$d_1 = 0.1\lambda$	$d_1 = 0.2\lambda$
$d_2 = .225\lambda$	$d_2 = .45\lambda$
$L_1 = .34\lambda$	$L_1 = .172\lambda$
$L_2 = .43\lambda$	$L_2 = .323\lambda$
$L_1' = .10\lambda$	$L_1' = .103\lambda$
$L_2' = .22\lambda$	$L_2' = .177\lambda$

Quarter wavelength Transformer

- It is simple and used for matching a real load to a TL.
- It can be extended to multisection design for broader bandwidth. But narrow band matching, a single section transformer shown in Figure is enough .



- The characteristic impedance of the matching section is $Z_l = \sqrt{Z_o R_L}$
- Since the length of the matching section is at the operating frequency , the perfect match cannot be obtained at other frequencies. The reflection coefficient at the input of the transformer can be determined as

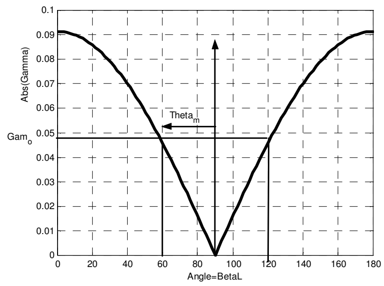
$$\Gamma = \frac{R_L - Z_o}{R_L + Z_o + j2\tan\beta l\sqrt{R_L Z_o}}$$

Near the design frequency the amplitude of the reflection coefficient is reduced to

$$|\Gamma| \cong \frac{|R_L - Z_o|}{2\sqrt{Z_o R_L}}|\cos\theta| \quad \text{for } \theta = \beta l \quad \text{near } \pi/2 .$$

Its variation as a function of angle is shown in Figure

The reflection coefficient amplitude for a single section quarter wave transformer near design frequency (R_L/Z_0=1.2)



f we set a maximum value for the reflection coefficient, say Γ_m, that can be tolerated, e can define bandwidth of the transformer as

$$\Delta\theta = 2\left(\frac{\pi}{2} - \theta_m\right)$$

hich is shown in Figure . The fractional bandwidth is found as

$$\frac{\Delta f}{f_o} = 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\frac{2\sqrt{Z_o R_L}}{|R_L - Z_o|}\right]$$

It is usually expressed as percentage, $100\Delta f / f_o\%$.

Example: Design a single section quarter wave transformer to match a 10Ω load to a 50Ω line at $f_0 = 3GHz$. Determine the percentage bandwidth for which the $VSWR \le 1.5$.

Solution:

The characteristic impedance of the matching section is

$$Z_1 = \sqrt{Z_o R_L} = 22.36\Omega .$$

The length of the matching section is $\frac{\lambda}{4} = 17.5cm$ at 3 GHz. An VSWR of 1.5 corresponds to a reflection coefficient magnitude of

$$\Gamma_m = \frac{VSWR - 1}{VSWR + 1} = 0.2 .$$

The fractional bandwidth is

$$\frac{\Delta f}{f_o} = 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\frac{2\sqrt{Z_o R_L}}{|R_L - Z_o|}\right] = 0.29, \textbf{ or } 29\%$$