# Measurement of the permittivity

Determining the complex dielectric constant of material samples from reflection coefficient measurements

## Fundamentals

The interaction of the transparent materials is characterized by complex permittivity

$$\underline{\varepsilon}_r = \varepsilon_r' - j\varepsilon_r'' = \varepsilon_r' (1 - j \tan \delta_\varepsilon) \qquad (9.1)$$

($\underline{\varepsilon}_r'$ = real component of $\underline{\varepsilon}_r$, tan $\delta_\varepsilon$ = dielectric loss factor) and the complex permeability

$$\underline{\mu}_r = \mu_r' - j\mu_r' = \mu_r' \ (1 - \mathrm{j} \tan \delta_\mu). \qquad (9.2)$$

These material parameters are generally dependent on the frequency.

For various technical problems and their solutions knowledge of these material parameters are required:

(a) If the materials are used for the construction of microwave components, then the exact values of $\underline{\varepsilon}_r$ and $\underline{\mu}_r$ are required as input data for computer codes in computer aided design (CAD). Examples: low-loss dielectrics as substrate materials for planar circuits, for example, in microstrip technology and for the realization of dielectric resonators for oscillator stabilization (DRO), and strongly absorbing materials for attenuating elements.

(b) Materials for the construction of radomes for antenna systems and absorber materials for reducing the radar cross-section of targets.

(c) The complex dielectric constant can frequently be used for the determination of another physical quantity. Thus the moisture content of a material, for example, (sand, coal, chipboards, tobacco etc.) modifies the real as well as the imaginary part of the effective dielectric constant. Consequently, it is possible to continuously monitor moisture content using microwaves.

(d) Should the material be heated with microwaves (cooking, drying, etc.), the complex dielectric constant and its change must be known, in order to optimize the corresponding microwave system.

(e) In physics and chemistry one can draw conclusions as to material composition from the position and width of the “absorption spectra” in the frequency response of $\underline{\varepsilon}_r$ or $\underline{\mu}_r$. If a material sample of a given shape is placed in a waveguide, the desired information on the value of $\underline{\varepsilon}_r$ or $\underline{\mu}_r$ is generally contained in the measurable reflection coefficient and transmission coefficient. For this the following considerations apply:

$\alpha$) Only for a geometrically “simple” sample does a simple mathematical expression apply for the relationship between the measurable (reflection coefficient and eventually the transmission coefficient) and the sought after quantities $\underline{\varepsilon}_r$ and $\underline{\mu}_r$

$\beta$) The dimensions of the samples and the selection of the measurement quantities must ensure a sufficient “sensitivity” on behalf of the measurement quantities to changes in the material parameters.

$\gamma$) If both $\underline{\varepsilon}_r$ as well as $\underline{\mu}_r$ are unknown, at least two independent (complex) measurement quantities must be determined. It is frequently known beforehand that the material has a value of $\underline{\mu}_r = 1$. In this case one measurement quantity would be sufficient for the determination of $\underline{\varepsilon}_r$.

In the following we shall presuppose that $\underline{\mu}_r = 1$, so that only $\underline{\varepsilon}_r$ needs to be determined.

Figure 9.1a shows the possible measurement configuration in which a material sample with the length $L$ with an unknown value of the (complex) relative permittivity $\underline{\varepsilon}_r$ fills up the rectangular waveguide completely.

The short-circuit plane coincides with the backside of the material sample for the position $l = 0$ of the moveable short , and thus the equivalent circuit diagram (b1) of the transmission line applies. Thus the following is true for the normalized input impedance:

$$\frac{\underline{Z}_A}{Z_0} = \sqrt{\frac{1-\left(\frac{\lambda_0}{2a}\right)^2}{\underline{\varepsilon}_r - \left(\frac{\lambda_0}{2a}\right)^2}} \cdot j \tan\left[\frac{2\pi L}{\lambda_0}\sqrt{\underline{\varepsilon}_r - \left(\frac{\lambda_0}{2a}\right)^2}\right] \qquad (9.3)$$

$Z_A/Z_0$ can be determined from the reflection measurements and $\lambda_0/2a$ as well as $L/\lambda_0$ are known variables. Consequently, the sought-after value of $\underline{\varepsilon}_r$ can be determined from Equation (9.3). However, Equation (9.3) is a complex transcendental equation, which can only be solved numerically ("zero position search program") for $\underline{\varepsilon}_r$.
Thus, a different approach should be taken here and, in addition, the normalized input impedance should be used for the open-circuit case. We then obtain the open-circuit case for the position $l = \lambda_g/4$ of the moveable short, and with the equivalent circuit according to (b2) in Figure 9.1 the result is

$$\frac{\underline{Z}_A}{Z_0} = \sqrt{\frac{1-\left(\frac{\lambda_0}{2a}\right)^2}{\underline{\varepsilon}_r-\left(\frac{\lambda_0}{2a}\right)^2}} \cdot \frac{1}{j\tan\left[\frac{2\pi L}{\lambda_0}\sqrt{\underline{\varepsilon}_r-\left(\frac{\lambda_0}{2a}\right)^2}\right]} \qquad (9.4)$$

If you form the product from $Z_A$ and $Z_B$, then the tan-function cancels out, and you obtain an expression which can be solved directly for the desired value $\underline{\varepsilon}_r$.

$$\underline{\varepsilon}_r = \left(\frac{\lambda_0}{2a}\right)^2 + \Lambda \cdot \left(1-\left(\frac{\lambda_0}{2a}\right)^2\right) \qquad (9.5)$$

whereby $\Lambda$ is used as an abbreviation for the expression

$$\Lambda = \frac{Z_0^2}{\underline{Z}_A \cdot \underline{Z}_B} \qquad (9.6)$$

$Z_A$ and $Z_B$ can be calculated from the values $r_A$ and $r_B$ of the reflection coefficient determined using the slotted measuring line via

$$\frac{\underline{Z}_A}{Z_0} = \frac{1+\underline{r}_A}{1-\underline{r}_A} \quad \text{and} \quad \frac{\underline{Z}_B}{Z_0} = \frac{1+\underline{r}_B}{1-\underline{r}_B} \qquad (9.7)$$

or determined using the Smith chart.

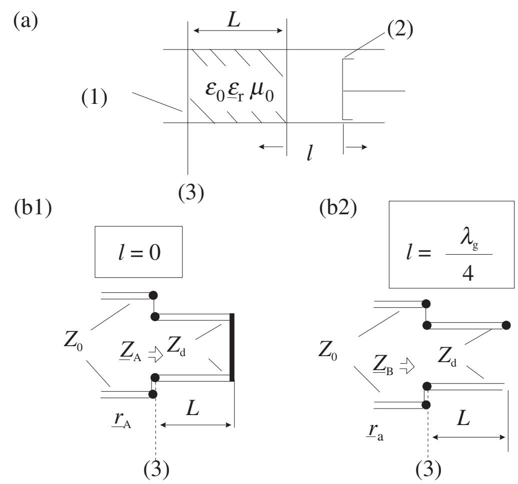


Fig. 9.1: (a) Measurement configuration with material sample (1) with the length $L$ and moveable short (2) connected downstream.
Distance of the short-circuit plane from rear side of material sample is $l$.
Reference plane for measurement is (3).
(b) Equivalent circuit diagram for trans. line for computation of the input impedances $Z_A$ and $Z_B$ for the short-circuit ($l = 0$) and open-circuit case ($l = \lambda_g/4$).

*Required equipment*

| | |
|---|---|
| 1 Basic unit | 737 021 |
| 1 Gunn oscillator | 737 01 |
| 1 Diaphragm with slit 2 x 15 mm, 90° | 737 22 |
| 1 Isolator | 737 06 |
| 1 PIN modulator | 737 05 |
| 1 Slotted measuring line | 737 111 |
| 1 Coax detector | 737 03 |
| 1 Short-circuit plate | 737 29 |
| 1 Sample holder | 737 29 |
| 1 Material sample of polystyrene | 737 29 |
| 1 Material sample of graphite | 737 29 |
| 1 Moveable short | 737 10 |
| 1 Set of thumb screws (2 each) | 737 399 |

*Additionally required equipment*

| | |
|---|---|
| 1 Oscilloscope (optional) | 575 29 |
| 1 XY recorder (optional) | 575 663 |
| 3 Coax cables with BNC/BNC plugs, 2 m | 501 022 |
| 2 Stand bases | 301 21 |
| 2 Supports f. waveguide components | 737 15 |
| 1 Stand rod 0.25 m | 301 26 |
| 1 Caliper | |

***Note:***

- To be able to obtain reproducible results, you need to use the PIN modulator with isolator, because in this experiment monomode operation is indispensible. Reproducible results can only be guaranteed with a PIN modulator.

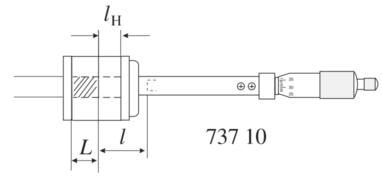


Fig. 9.2: Measurement object for Experiment 9

## Experiment procedure

*1. Setup and calibration measurement*

1.1 Set up the experiment according to Fig. 9.3. Arrange the measurement object according to Fig. 9.1 and Fig. 9.2.

1.2 Attach the short-circuit plate to the open end of the slotted measuring line.

1.3 Determine the position $x_0$ of the first minimum ("from the left") on the slotted measuring line and enter the value into Table 9.1. Determine the position $x_1$ of the 3rd minimum and enter it into Table 9.1.

1.4 Calculate the guided wavelength

$$\lambda_g = |x_0 - x_1| .$$

*2. Determining short-circuit and open-circuit input reflection coefficient for sample I (polystyrene)*

2.1 Insert sample I (polystyrene, black) into the sample holder. On the positioning of the sample in the holder please refer to Fig. 9.2. Use a caliper to measure the distance $l_H$ to the "right-hand" edge of the sample holder ($l_H$ should amount to just about 10.0 mm exactly).

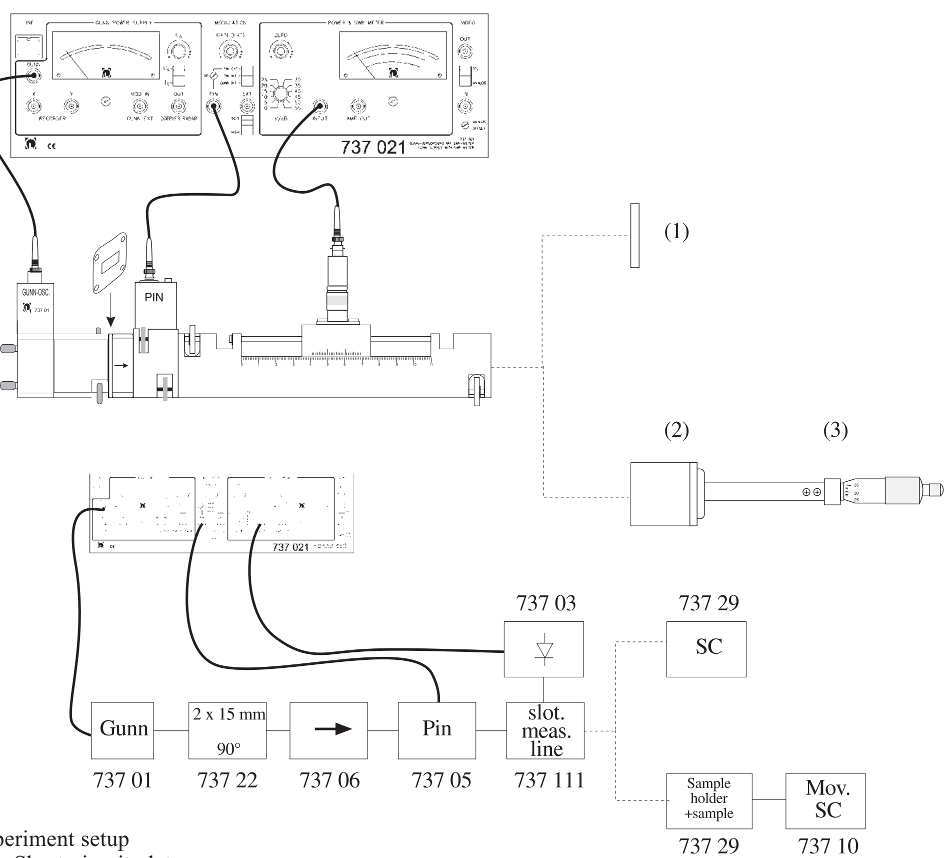


Fig. 9.3: Experiment setup
(1) Short-circuit plate
(2) Meas. object corresponding to Fig. 9.2 (see also Fig. 9.1)
(3) Moveable short-circuit

2.2 Attach the sample holder (with sample) to the open end of the slotted measuring line (see Fig. 9.3).
2.3 Attach the moveable short to the open side of the sample holder.
2.4 Adjust the moveable short ($x_k$) so that the position of the short-circuit plane coincides with the rear side of the sample ($l = \lambda_g/2$, i.e. $x_k = \lambda_g/2 - l_H$). Note: Read $x_k$ from the micrometer of the variable short.
2.5 Based on the standing wave ratio $s_A$ and the position of the minimum $x_A$ (first from the “left”), determine the short-circuit reflection coefficient:

$$\underline{r}'_A = |\underline{r}_A| e^{+j\phi'_A}$$

according to magnitude and phase (reference plane = front surface of the sample). Enter into Table 9.1. (Note: see Table 9.1 for the individual mathematical steps)
2.6 Shift the moveable short further by one quarter of a guided wavelength ($\lambda_g/4$) so that the open-circuit plane coincides with the rear side of the sample.
2.7 Based on the standing wave ratio and the position of the minimum determine the open-circuit reflection coefficient

$$\underline{r}'_B = |\underline{r}'_B| e^{+j\phi'_B}$$

according to magnitude and phase and enter the values into Table 9.1.

3. *Determining the short-circuit and open-circuit input reflection coefficient for sample II. (non-magnetic absorber material made of synthetic resin with graphite, color: anthracite )*
Analogous to 2.1 to 2.7 determine

$$\underline{r}''_A = |\underline{r}''_A| e^{+j\phi''_A}$$

and

$$\underline{r}''_B = |\underline{r}''_B| e^{+j\phi''_B}$$

Enter the values into Table 9.1.

***General notes***

- To avoid big mistakes when determining the dielectric constant or permittivity, the phase measurements have to be performed with great care (precise values of $x_A$, $x_B$ and $x_0$). Furthermore, the guided wavelength must be known precisely (determine using the slotted measuring line) and the moveable short ($x_k$) must be set precisely (so that $l = \lambda_g/2$ or $3\lambda_g/4$, see Fig. 9.1).
- Tip: For the determination of the minimum, the gain should be successively increased to find the location with more precision. An even finer setting is possible by a slight tapping, i.e. moving the carriage with a pen.

## Questions

1. In accordance with the equations

$$\frac{\underline{Z}_A}{Z_0} = \frac{1+\underline{r}_A}{1-\underline{r}_A} \quad \text{and} \quad \frac{\underline{Z}_B}{Z_0} = \frac{1+\underline{r}_B}{1-\underline{r}_B}$$

calculate the normalized short-circuit and open-circuit impedances for samples I and II and enter the values into Table 9.1. (Alternatively: Determine using the Smith chart, see Fig. 9.4).

2. Calculate the expression (see also Equation 9.6)

$$\Lambda = \frac{Z_0^2}{\underline{Z}_A \cdot \underline{Z}_B}$$

for samples I and II and enter the values into Table 9.1.
3. Determine the complex dielectric constants from sample I and II according to

$$\underline{\varepsilon}_r = \left(\frac{\lambda_0}{2a}\right)^2 + \Lambda \cdot \left(1 - \left(\frac{\lambda_0}{2a}\right)^2\right)$$

($\lambda_0$= free-space wavelength,
$a$ = waveguide width = 22.8 mm).
Enter the final results into Table 9.1.

*Table 9.1*

$x_0$ = ________ mm $x_1$ = ________ mm

$\lambda_g$ = ________ mm ⇒ $\lambda_0$ = ________ mm

| | Sample I (Polystyrol) | Sample II (Absorber material) |
|---|---|---|
| VSWR $s_A$ | | |
| $x_A$/mm | | |
| $\lvert \underline{r}_A \rvert = \frac{s_A - 1}{s_A + 1}$ | | |
| $\phi_A = 180° - 720° \cdot \frac{x_A - x_0}{\lambda_g}$ | | |
| $\underline{r}_A = \lvert \underline{r}_A \rvert \cdot (\cos\phi_A + j \cdot \sin\phi_A)$ | | |
| from the Smith chart or mathematically $\frac{\underline{Z}_A}{Z_0} = \frac{1+\underline{r}_A}{1-\underline{r}_A}$ | | |
| VSWR $s_B$ | | |
| $x_B$/mm | | |
| $\lvert \underline{r}_B \rvert = \frac{s_B - 1}{s_B + 1}$ | | |
| $\phi_B = 180° - 720° \cdot \frac{x_B - x_0}{\lambda_g}$ | | |
| $\underline{r}_B = \lvert \underline{r}_B \rvert \cdot (\cos\phi_B + j \cdot \sin\phi_B)$ | | |
| from the Smith chart or mathematically $\frac{\underline{Z}_B}{Z_0} = \frac{1+\underline{r}_B}{1-\underline{r}_B}$ | | |
| $\Lambda = \frac{Z_0^2}{\underline{Z}_A \cdot \underline{Z}_B}$ | | |
| $\underline{\varepsilon}_r = \varepsilon'_r - \varepsilon_r''$ | | |
| $\tan\delta_\varepsilon = \varepsilon_r''/\varepsilon_r'$ | | |

Smith Chart

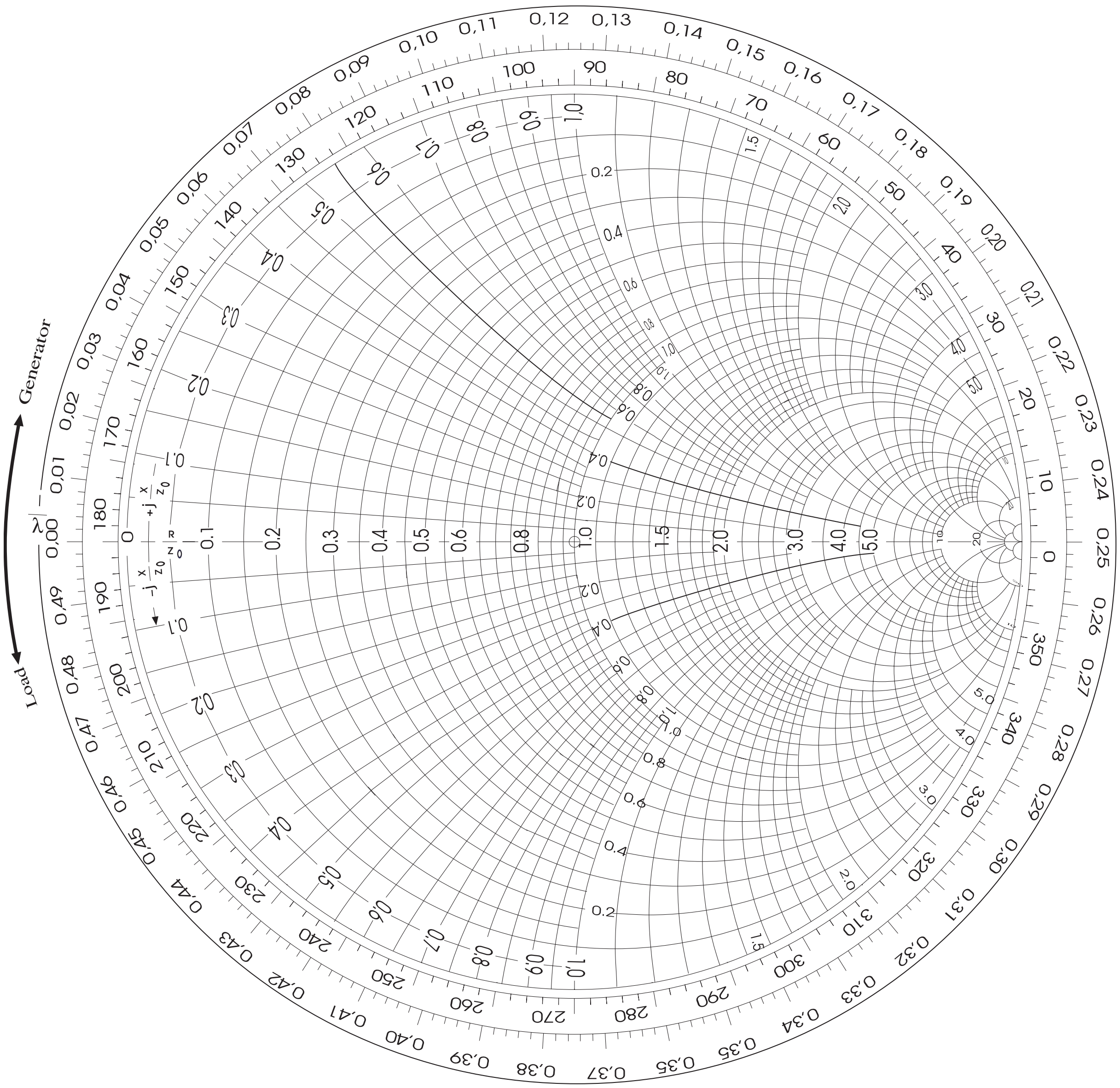


Fig. 9.4: Smith chart

## Bibliography

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[3] F. E. Gardiol: Introduction to Microwaves. Artech House, Dedham MA, 1984