Matching and the Smith chart

Principles

Preliminary considerations

If the input impedance Z of a one-port is not in agreement with the characteristic impedance Z_0 of the transmission line this results in a reflection characterized by the reflection coefficient

$$\underline{r} = |\underline{r}| \cdot e^{j\phi} = \frac{\left(\frac{Z}{Z_0}\right) - 1}{\left(\frac{Z}{Z_0}\right) + 1} \qquad (7.1)$$

For various reasons this reflection is undesirable for most applications. Some of the reasons are listed here:

(a) If the transmission line is fed from a generator with the available power P_{av} and this generator is matched to the line (see Figure 7.1, part 1), then the real power absorbed by the load is

$$P = P_{av}\,(1 - |r|^2). \qquad (7.2)$$

This shows that the maximum power is only supplied to the load when it is perfectly matched; ($|r| = 0$).

(b) Because of non-linear effects, the existence of a wave returning to the generator can lead to changes in the operation characteristics of the generator, e.g. to a frequency shift and to parasitic oscillations at different frequencies (fulfilling the condition for self-excitation through reflection).

(c) Compared with the case of $|r| = 0$ an enhanced field strength (standing wave) is caused by the interference of the reflected wave with the one travelling forward. Thus, the danger of disruptive discharges (high electric field strengths) is associated with the transport of relatively high power levels.

The danger described in point (c) does not exist in this training system because only low power levels are used here.

Basic function of a lossless matching element and the matching condition

For a given frequency one can generally match any one-port with $|r| < 1$ to the characteristic impedance Z_0 by positioning a two-port (= matching network) upstream in series between the reflecting one-port and the transmission line. If a lossless two-port (ideally) is used for the matching network, then its general function is to compensate the given reflection coefficient r to zero by adding additional reflections. Section 2 of Fig. 7.1 initially demonstrates the general situation. By connecting the network N (two-port) in series upstream, the reflection coefficient r is transformed into the reflection coefficient $\underline{\tilde{r}}$. The aim is to achieve correct matching by selecting the right parameters of the lossless linear two-port, i.e. to achieve $\underline{\tilde{r}} = 0$.

The condition to be fulfilled here (matching condition) can be deduced in general, that means without any special knowledge regarding the construction of the two-port N. Here a consideration of only one parameter of the two-port is sufficient; namely the backward reflection coefficient $\underline{\Gamma}$, the definition of which is shown in section 3 of Fig, 7.1. Here, it is assumed that port 1 (1-1') of the two-port is terminated reflection-free (i.e. the internal impedance of the source is Z_0). $\underline{\Gamma}$ is then the effective reflection coefficient "felt" by a wave travelling to port 2 (2-2') from right to left. (Bear in mind that r applies to a wave arriving from the left.)

You can demonstrate that it is sufficient for matching when $\underline{\Gamma}$ is chosen to be the conjugate complex value of of the reflection coefficient r to be matched (see below):

$$\underline{\Gamma} = r^* = |r|\, e^{-j\phi} \text{ (matching condition)} \qquad (7.3)$$

Section 4 of Fig. 7.1 shows the resulting situation for this specific case. At port 1 the reflection results in $\underline{\tilde{r}} = 0$ (matching). Between port 2 and the mismatched ($|r| \neq 0$) one-port you obtain the superpositioning of a wave with the power $P_{av}/(1 - |r|^2)$ travelling to the one-port while the reflected wave has the power $P_{av} \cdot |r|^2/(1 - |r|^2)$.

The total power results from the difference of the power of both waves and equals P_{av}. Thus, as desired, the total available power is provided to the one-port (for a lossless matching network).

Matching element according to the principle of the slide screw transformer

According to the findings from the previous section there are two different ways to explain how a matching element works:

(A) The explanation dealing with the transformation of a given reflection coefficient $\underline{r} = |\underline{r}|\, e^{j\phi}$ to the value $\underline{\tilde{r}} = 0$ using a matching element.

(B) The explanation involving the setting $\underline{\Gamma} = |\underline{r}|\; e^{-j\phi}$ of the matching element connected reflection-free at the input-side.

According to Figure 7.2 a slide screw transformer consists of a homogeneous waveguide section designed with an “obstacle” which can be adjusted in terms of location (variable position setting ξ_0) and “magnitude”. If the “obstacle” is in the form of a metal post as in Figure 7.2 (left) with an adjustable penetration depth of $h < b$, this can be represented in the equivalent circuit diagram by a shunt capacitance C (see Fig. 7.2, right) as long as h is sufficiently small compared to the height b of the waveguide. C is zero for $h = 0$ and increases with h. If h is only “slightly” smaller than b, one obtains a series resonant circuit, whereas, if the post touches the opposite side, it responds like a shunt inductance. In the case of the slide screw transformer here we can always assume that we are dealing with a shunt capacitance.

Now we shall first consider the transformation of a random reflection coefficient $\underline{r} = |\underline{r}|\, e^{j\phi}$ in the

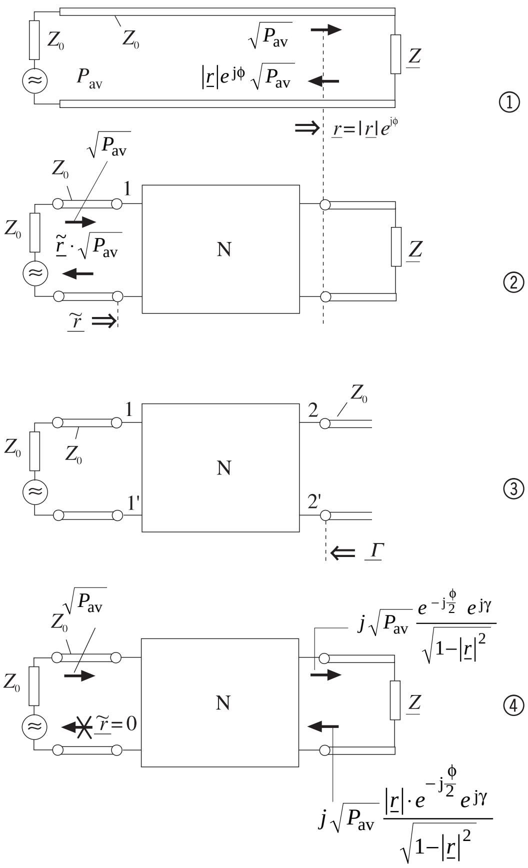

Fig. 7.1: General considerations regarding the problem of matching

① Transmission line with matched generator and mismatched load. Corresponding complex amplitudes from wave travelling to and reflected from load.

② Series connected two-port transforms reflection coefficient r into the value $\tilde{r}$.

③ On the definition of the backwards reflection coefficient G of the two-port

④ Ratios for matching, i.e. two-port N is the ideal non-dissipative matching network.

f: Phase displacement through Z

g: Phase displacement through the matching network

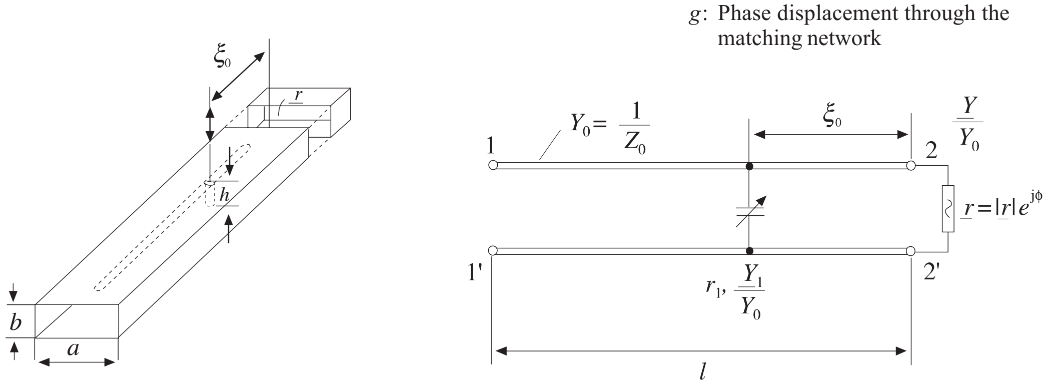

Fig. 7.2: Technical construction of a slide screw transformer (left) and corresponding equivalent circuit diagram (right).

matching point $\underset{\sim}{r} = 0$ (Explanation A according to above case distinction).

In Fig. 7.3 point 1 specifies the random reflection coefficient $r = |r|\, e^{j\phi}$ (in the example: $|r| = 0.605$ and $\phi = 210°$). To be able to read the admittances out of the Smith chart (expedient for parallel circuits), the transition to $-r$ (point 2) is carried out through inversion at the matching point ($r = 0$). In the example $Y/Y_0 = 2 + j\,1.9$ of the normalized admittance belongs to the value of $-r$ (for practice please verify in Fig. 7.3).

Based on this preliminary step it is now easy to explain the function of the slide-screw transformer. Only the phase of the reflection coefficient but not the magnitude is changed by the waveguide section with the length ξ_0 (phase rotation by the angle, $720° \; \xi_0 / \lambda_g$ in clockwise

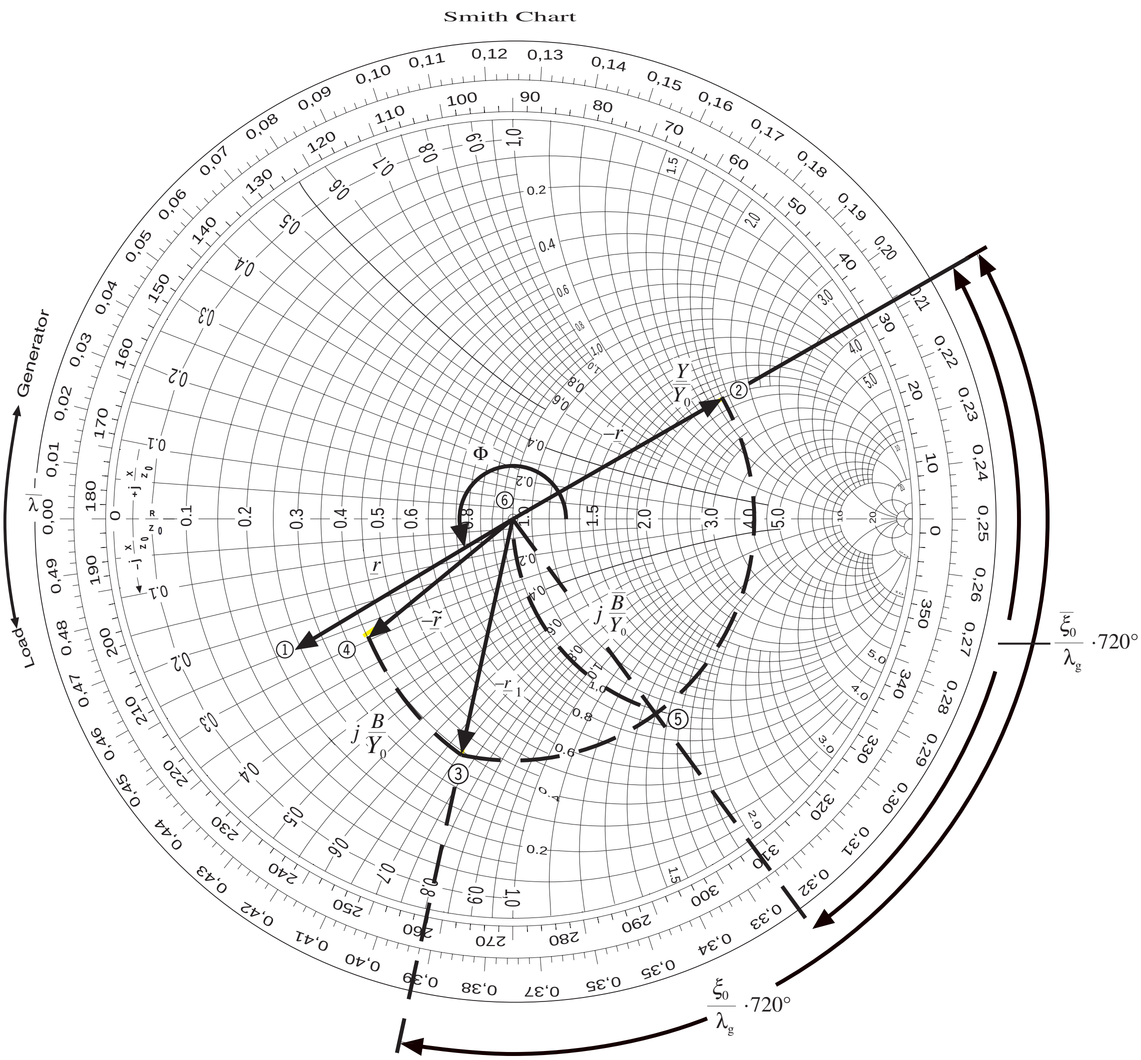

Fig. 7.3: Matching of a random reflection coefficient $r = |r|\, e^{j\phi}$ (where $|r| < 1$) with the aid of a slide screw transformer:
① Reflection coefficient r to be matched
② Point for $-r$ (admittance representation)
③ Negative reflection coefficient $-r_1$ after transformation by the waveguide section of the arbitrary length ξ_0.
④ Negative reflection coefficient $-\underset{\sim}{r}$ (resp. $\underset{\sim}{Y}/Y_0$) after parallel connection of a positive susceptance B of random magnitude.
Matching case: ⑤ and ⑥ represent $-r_1$ and $-\underset{\sim}{r}$ for the correct selection $\xi_0 = \overline{\xi}_0$ and $B = \overline{B}$ for the line length and susceptance.

direction). Thus, one obtains the value of $-r_1$ given by point 3 in Figure 7.3. The parallel connection of $j \cdot B$ corresponds to the addition of an imaginary admittance and consequently the value of the reflection coefficient changes into $\tilde{\underline{r}}$ (point 4) along the locus of a constant real component (in this example Re $(Y/Y_0) = 0.4$).
In order to achieve matching, i.e. $r = 0$, the length $\xi_0 = \overline{\xi_0}$ must be selected in accordance with Figure 7.3 so that $-r_1$ is located on the locus Re $(Y/Y_0) = 1$ (point 5 where $\overline{\xi_0} \approx 0.118 \cdot \lambda_g$). Under this prerequisite there exists a value $B = \overline{B}$ for which $-\tilde{\underline{r}}$ is located at the matching point (point 6) (in the example: $\overline{B}/Y_0 \approx 1.5$).
Predicated on the explanations and descriptions found in the previous section an alternative explanation suggests itself for the function of the slide screw transformer (explanation B according to the case distinction above). In this presentation we will be referring to the circuit found on the left of the reference plane (Symbol $\underline{\Gamma}$) and not to the circuit found on the right – as above. If the slide screw transformer is terminated reflection-free at port (1- 1' in Fig. 7.4), then $\tilde{\underline{\Gamma}} = 0$ (see Fig. 7.4) also applies to the left of the shunt capacitance. $\underline{\Gamma}_1$ is obtained to the right of the shunt capacitance C, whereby the magnitude $|\underline{\Gamma}$€$_1|$ can be altered by varying C (penetration depth of the post) between 0 and values "approximating" 1. Here the phase of $\underline{\Gamma}$€$_1$ also changes, but as a function of the magnitude $|\underline{\Gamma}$€$_1|$. A phase variation independent of the magnitude is possible by changing the location ξ_0. As such the phase condition for matching specified above

$$\underline{\Gamma}€ = r^* = |r|\, e^{-j\phi}$$

can always be fulfilled by varying C and ξ_0.

Matching element according to the principle of the multi-screw transformer (2- or 3-screw transformer)
There are two variable parameters in the slide screw transformer dealt with above, namely the position ξ_0 and the penetration depth h of the post (screw). In the case of multi-screw transformers we dispense with an adjustment of the screw position (x-coordinate). To realize this the number of screws is greater than 1 and each of the two or three screws can be adjusted independently of each other in terms of their penetration depth.
The lower part of Fig. 7.5 shows the equivalent circuit diagram for a 2-screw transformer. The positions (ξ_0 and $\Delta\xi$) of the screws are fixed but the penetration depth and therefore both susceptances (B_1 and B_2) can be adjusted independently.
The negative value $-r$ of the reflection coefficient at the end of the transmission line (r) is given by point 1 (bear in mind: the same value as in Fig. 7.3). The waveguide section of the fixed length ξ_0 transforms $-r$ into point 2 (same magnitude, phase rotated by $[\xi_0/\lambda_g] \cdot 720°$ in the clockwise direction). By means of a parallel connection of $j\,B_1$ one obtains a value corresponding to point 3. The waveguide section of fixed length $\Delta\xi$ transforms point 3 into point 4 (same magnitude, rotation of the phase by $(\Delta\xi/\lambda_g) \cdot 720°$ in the clockwise direction) and finally the parallel connection of $j\,B_2$ transforms point 4 into point 5. In order for point 5 to be located in the matching point ($r = 0$), B_1 must be

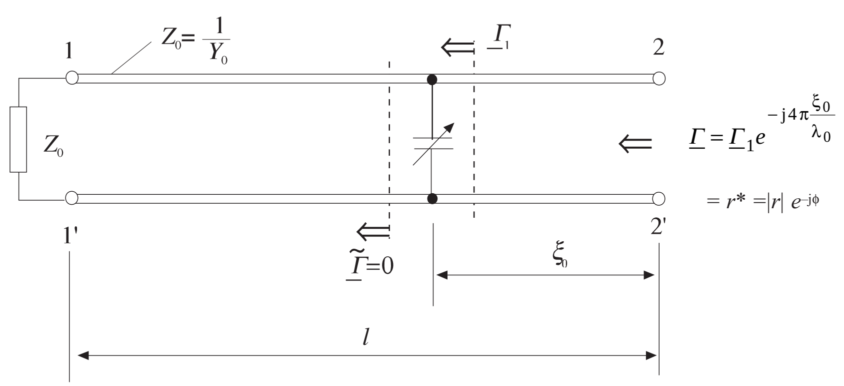

Fig. 7.4: How the slide screw transformer works (alternative explanation to Fig. 7.3).

selected precisely so that point 3 is located on the auxiliary circle *K'*. The auxiliary circle *K'* is obtained from circle *K* (Re $\{Y/Y_0\} = 1$) by rotation around the matching point (rotation angle $[[\Delta\xi/\lambda_0] \cdot 720°$ counterclockwise). As you can conclude from Figure 7.5, not every reflection coefficient $|r| < 1$ can be matched using the 2-screw transformer.

However, this can be achieved using the 3-screw configuration. But in this case there are several solutions for B_1, B_2 and B_3 for a given value of *r*. Consequently, we shall dispense with a more detailled theoretical investigation of the 3-screw transformer.

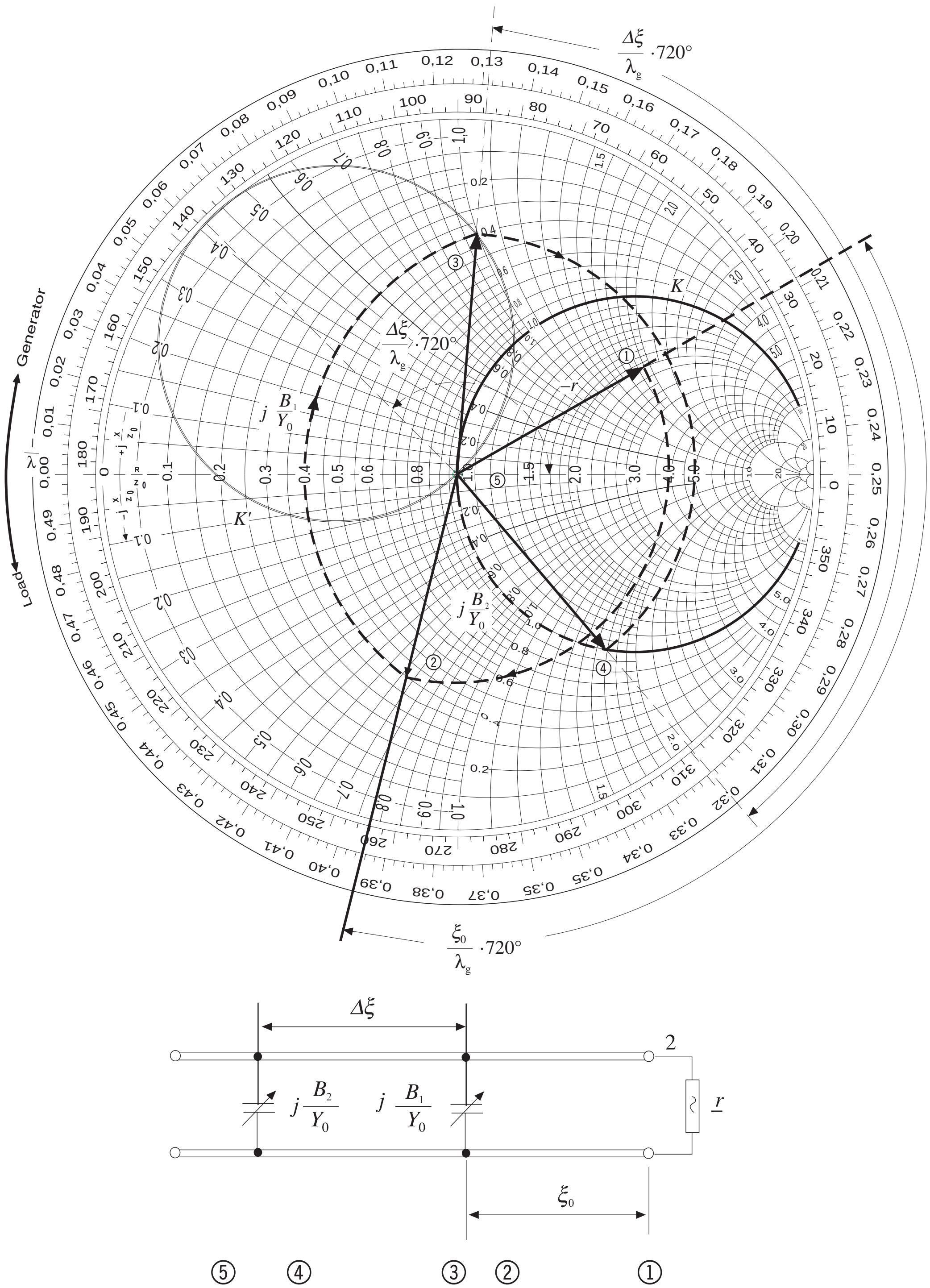

Fig. 7.5: Matching using a 2-screw transformer.

Required equipment

1 Basic unit	737 021
1 Gunn oscillator	737 01
1 Diaphragm with slit 2 x 15 mm 90°	737 22
1 Isolator	737 06
1 PIN modulator	737 05
1 Slotted measuring line	737 111
1 Coax detector	737 03
1 Cross directional coupler	737 18
1 Transition waveguide / coax	737 035
1 Waveguide 200 mm	737 12
1 3-screw transformer	737 135
1 Short-circuit plate, from accessories	737 29
1 Sample holder, from accessories	737 29
1 Graphite sample, from accessories	737 29
2 Waveguide terminations	737 14
1 Set of thumb screws (10 each)	737 399

Additionally required equipment

1 Oscilloscope (optional)	575 29
1 XY recorder (optional)	575 663
3 Coaxial cables with BNC/BNC plugs, 2 m	501 022
2 Stand bases	301 21
2 Supports for waveguide components	737 15
1 Stand rod 0.25 m	301 26

Recommended

1 Slide screw transformer	737 13
1 Set of thumb screws (2 each)	737 399

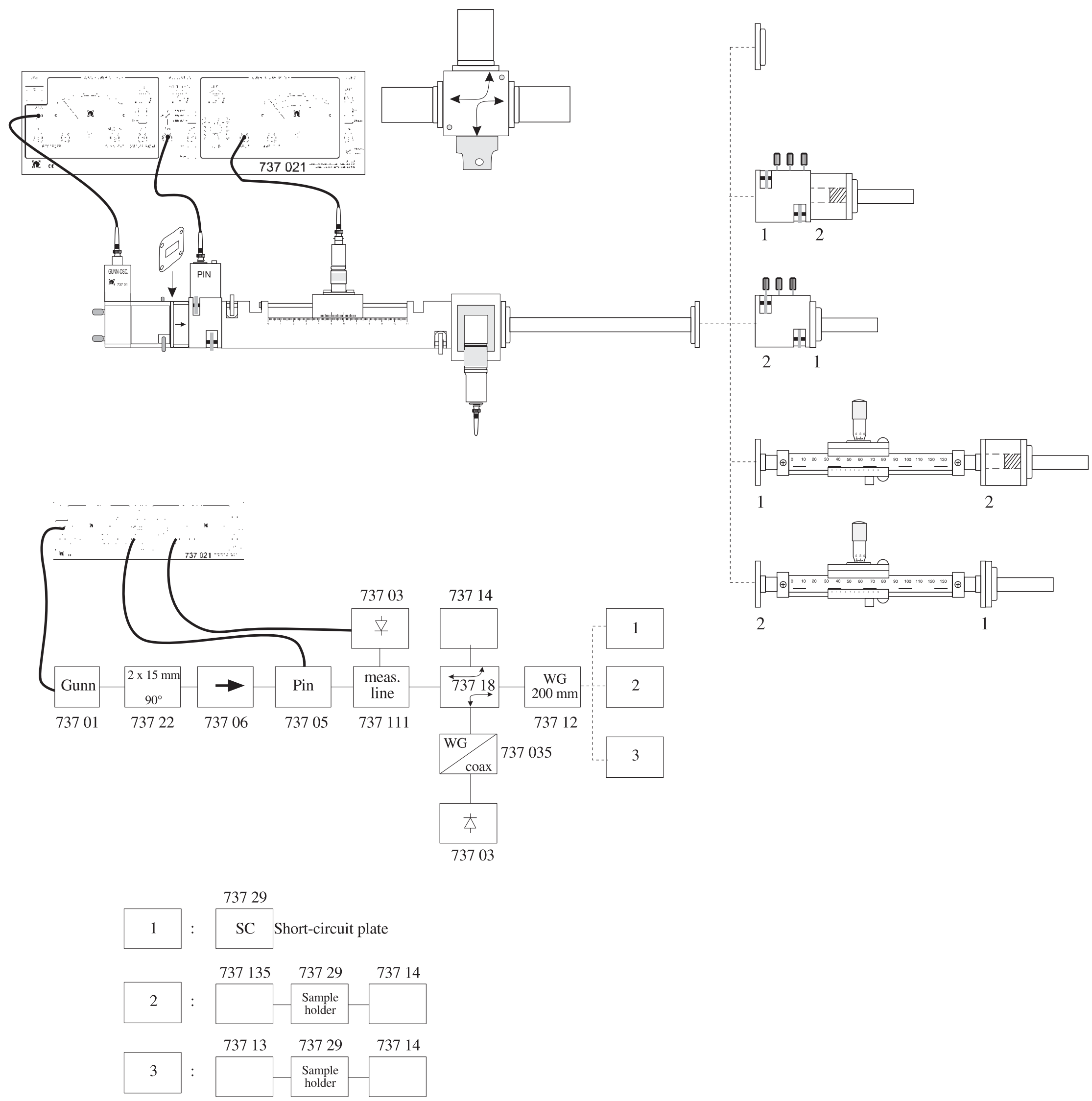

Fig. 7.6 Experiment setup

Notes:

- In the experiment a cross directional coupler is used (Coupling diaphragm with 2 cross-shaped holes) to measure the reflected wave. The cross directional coupler is an arrangement made up of 2 waveguides, which are connected together vis-à-vis coupling holes. A portion of the travelling and reflecting wave can be detected at the output ports of the coupling waveguide. The exact function of the cross directional coupler is explained in Experiment 10.
- The coax detector is connected alternately to the slotted measuring line and the cross directional coupler. Naturally, it is also possible and even recommended to use two detectors.
- Since this experiment responds with particular sensitivity to parasitic modes you can only expect reproducible results when using a PIN modulator (see also "Design of the Microwave Source" from the preface).

Experiment procedure

1. *Calibration*

1.1 Set up the experiment in accordance with Fig. 7.6. First attach the short-circuit plate to the open end of the line.

1.2 Use the slotted measuring line to determine the location x_0 of the first ("right") field strength minimum.
Enter this value into Table 7.1.

1.3 Replace the short-circuit plate with the measurement object A from Experiment 6, consisting of a sample holder with graphite sample and waveguide termination and determine the location of the first (again from the "right") minimum x_1/mm and enter the value into Table 7.1 a).

2. *Matching using the 3-screw transformer*

2.1 Insert the 3-screw transformer between the reflecting one-port (measurement object A from Experiment 6) and the "200 mm waveguide".

2.2 Connect the coax-detector to the cross directional coupler to measure a portion of the reflected microwave. Here be sure to select a suitable gain factor V/dB.

2.3 Make an attempt to achieve matching by adjusting all three screws one after the other in succession. This means a minimizing of the reflected microwave power (ideally: no reflected wave). Here the best procedure is to find the screw which has the greatest effect on the reflected wave. To do this turn one of the three screws (increasing the penetration depth) while keeping the other two respective screws set to a penetration depth of 0. Then set the screw with the greatest effect to its optimum setting (local minimum of the reflected wave) and then turn the other screws in succession to further diminish the reflected wave. At the same time you have to suitably adapt the gain factor to be able to determine the optimum setting of the matching element.
Note: If you are unable to obtain a distinct response for any of the screws, then it might prove useful to turn the 3-screw transformer in the configuration (i.e. interchanging the ports), because it is designed slightly asymmetrically. This gives you a different phase angle for the screws.

2.4 Now replace the coax detector again with the slotted measuring line and check the transforming effect on the standing wave ratio (reset the maximum to 0 dB). The standing wave ratio should tend towards the value 1 ($s \to 1$, i.e. $|r| \to 0$, ideally: $s = 1$, $|r| = 0$). You have attained good matching if you reach a standing wave ratio under $s < 1.1$. In order to rule out any influence of the cross directional coupler on the reflection coefficient, you can also connect the matching transformer plus the measurement object directly to the slotted measuring line (i.e. remove the cross directional coupler and the waveguide section).

2.5 Afterwards measure the backward reflection coefficient $\underline{\Gamma}$ of the 3-screw transformer without any further adjustment to the screws. To do this

(α) Remove the reflecting single-port.

(β) Reverse the connection of the 3-screw transformer (reversing the ports) to the slotted measuring line.

(γ) Equip the open end of the 3-screw transformer with the reflection-free waveguide termination (see also Fig. 7.6).

Read off the standing wave ratio s and the value of x_1. Enter the findings in Table 7.1 b).

3-screw transformer

Table 7.1

- Short-circuit plate x_0 = ________ mm

a) Measurement object without 3-screw transformer

VSWR s	Minimum at x_1/mm	$\lvert \underline{r} \rvert = \frac{s-1}{s+1}$	$\phi = 180° - 720° \cdot \frac{x_1 - x_0}{\lambda_g}$

b) Calibrate to $s \to 1$, i.e. $\lvert r \rvert \to 0$ by successively turning all the screws
Backward reflection coefficient $\underline{\Gamma}$ (see Fig. 7.1)

VSWR s	Minimum at x_1/mm	$\lvert \underline{\Gamma} \rvert = \frac{s-1}{s+1}$	$\phi_\Gamma = 180° - 720° \cdot \frac{x_1 - x_0}{\lambda_g}$

3. Matching using the slide screw transformer (optional)

3.1 In accordance with Fig. 7.6 insert the slide screw transformer between the reflecting one-port (measurement object A from Experiment 6) and "waveguide 200 mm". Connect the coax detector to the cross directional coupler.

3.2 Minimize the reflected wave by successively adjusting the longitudinal position *x'* and the penetration depth *h*. In general there are several combinations you can use to achieve matching.

3.3 Again mount the coax detector on the slotted measuring line and check the effect on the standing wave ratio (reset the maximum to 0 dB). You have again reached good matching if $s < 1.1$ (i.e. *r*€< 0.05). To rule out any influence from the cross directional coupler on the reflection coefficient, you can once again connect the matching transformer plus measurement object to the slotted measuring line (i.e. remove the cross directional coupler and the waveguide section).Use a table similar to table 7.1a.

3.4 The backward reflection coefficient $\underline{\Gamma}$ of the slide screw transformer is determined for matching. The measurement procedure is analogous to that shown in experiment point 2.5. Determine the standing wave ratio s and x_1. Use a table similar to Table 7.1b.

Questions

1. For measurement object A calculate the values of $r = |r|\, e^{\,j\phi}$ corresponding to the measurement data.

2. Determine the complex value $\underline{\Gamma}$ of the backward reflection coefficient of the 3-screw transformer based on the measurement data in Table 7.1 b). Check the validity of Equation (7.3).

(Optional)

3. Determine the complex value ($|\underline{\Gamma}|$ and ϕ_Γ) of the backward reflection coefficient from the measurement data. Test the validity of Equation (7.3) by comparing the value determined for $\underline{\Gamma}$ with the value of r. Compare the value determined for $\underline{\Gamma}$ using the slide screw transformer with the respective value determined using the 3-screw transformer.

Bibliography

See Experiment 5

Smith Chart

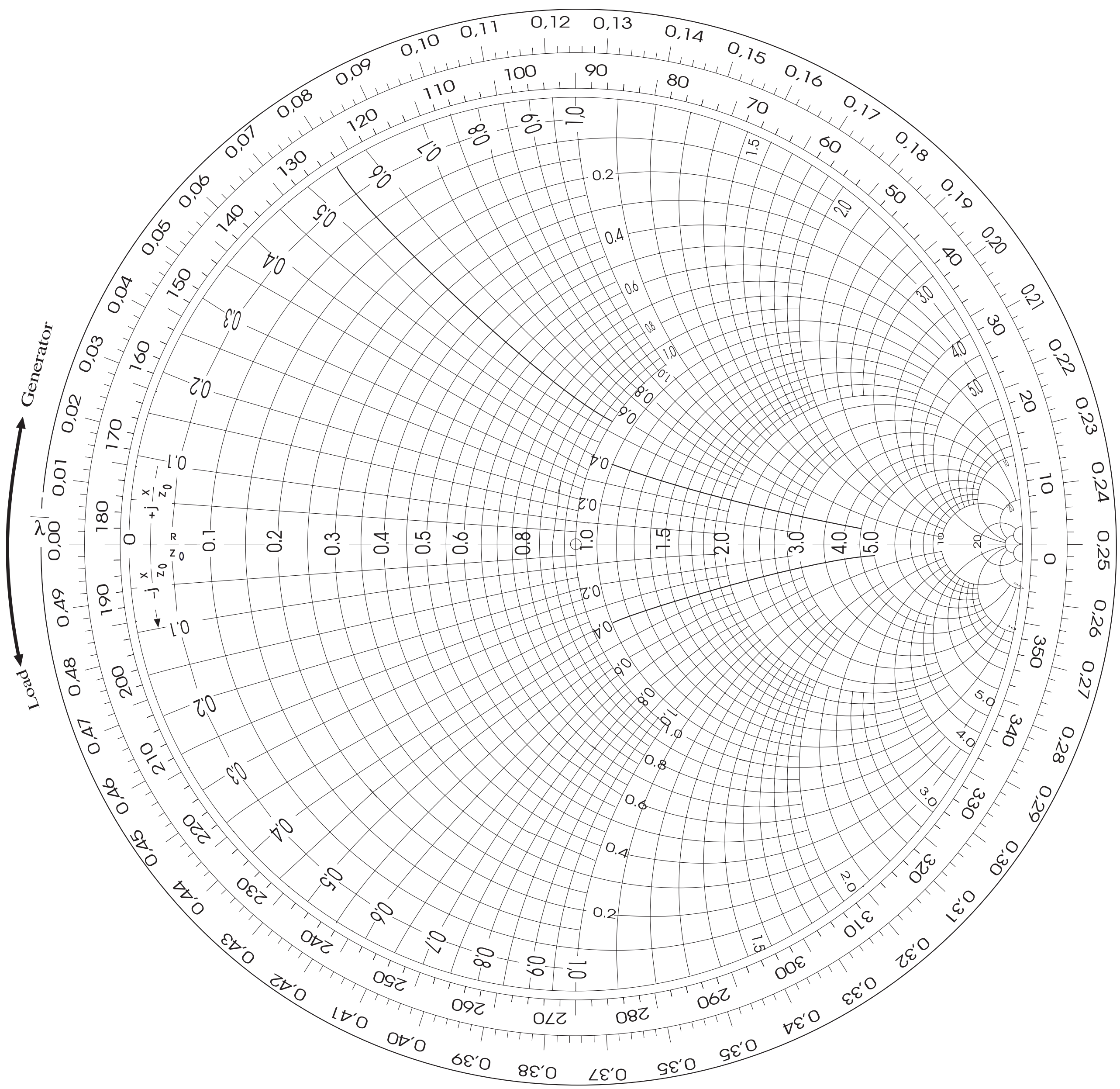

Fig. 7.7: Smith chart