The complex reflection coefficient

Determining the reflection coefficient according to magnitude and phase

Principles

Voltage curve for random termination impedance

In Experiment 5 two special cases were studied, namely the case where a line is terminated in a short-circuit ($r = -1$) and a line which is terminated with total absorbtion ($r = 0$). Now we will describe in more detail the standard case where the transmission line is generally terminated with any given complex impedance Z (see Fig. 6.1(a)).

In this case we have the superpositioning of two waves, a reflected wave $U_{0,-} \cdot e^{+j\beta x}$ travelling in the ($-x$) direction and the wave $U_{0,+} \cdot e^{-j\beta x}$ travelling in the ($+x$) direction, whereby the (complex) reflection coefficient r is defined as the ratio of the (complex) amplitudes of the wave being sent back and the wave travelling to the load at the point of reference (here $x = 0$):

$$\underline{r} = |\underline{r}| \cdot e^{j\phi} \overset{\text{def}}{=} \frac{\underline{U}_{0,-}}{\underline{U}_{0,+}} \qquad (6.1)$$

This results in a spatial dependency for the complex amplitude of total voltage $U(x)$ and total current $I(x)$ according to

$$\underline{U}(x) = \underline{U}_{0,+} \cdot \left[e^{-j\beta x} + \underline{r} \cdot e^{j\beta x}\right] \qquad (6.2a)$$

$$\underline{I}(x) = \frac{\underline{U}_{0,+}}{Z_0} \cdot \left[e^{-j\beta x} - \underline{r} \cdot e^{j\beta x}\right] \qquad (6.2b)$$

But due to the fact that the ratio $U(x)/I(x)$ at $x = 0$ must be equal to the impedance Z, it is true that

$$\underline{Z} = \frac{\underline{U}(0)}{\underline{I}(0)} = \frac{1+\underline{r}}{1-\underline{r}} \cdot Z_0 \qquad (6.3)$$

Solving equation (6.3) for r yields the significant relationship

$$\underline{r} = |\underline{r}| \cdot e^{j\phi} = \frac{\left(\frac{\underline{Z}}{Z_0}\right) - 1}{\left(\frac{\underline{Z}}{Z_0}\right) + 1} \qquad (6.4)$$

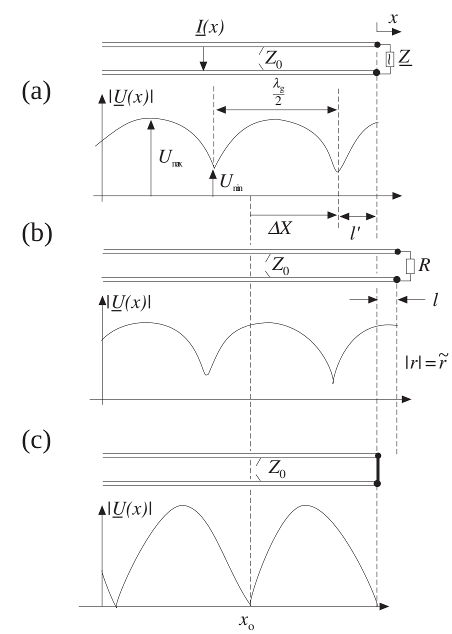

Fig. 6.1: Voltage distribution along a transmission line
(a) Random complex termination impedance Z
(b) Equivalent circuit for Z (R and line with a length of l)
(c) Shorted transmission line

If we consider the distribution of the magnitude $|U(x)|$ along the transmission line, then it follows from equation (6.2 a) that

$$|\underline{U}(x)| = |\underline{U}_{0,+}| \cdot \sqrt{1 + |\underline{r}|^2 + 2 \cdot |\underline{r}| \cos(2\beta \cdot x + \phi)} \qquad (6.5)$$

Part a of Fig. 6.1 shows the voltage distribution, where it is true that

$$U_{max} = |\underline{U}_{0,+}| \cdot (1 + |\underline{r}|) \text{ and}$$
$$U_{min} = |\underline{U}_{0,+}| \cdot (1 - |\underline{r}|) \qquad (6.6)$$

The voltage standing wave ratio (VSWR = s) is given by:

$$s \overset{\text{def}}{=} \frac{U_{max}}{U_{min}} = \frac{1+|\underline{r}|}{1-|\underline{r}|} \qquad (6.7)$$

and

$$\left|\underline{r}\right| = \frac{s-1}{s+1} \tag{6.8}$$

Determining the complex reflection coefficient from the measurements performed with the slotted measuring line

According to Eq. (6.8) the magnitude $|r|$ of the reflection coefficient can be calculated from the standing wave ratio ***s*** determinable using the slotted measuring line.
The phase ϕ of the reflection coefficient can be determined by comparing the location x_1 of the minimum value of $|U(x)|$ to the location $\boldsymbol{x_0}$ of the zero point or minimum when the transmission line is shorted.
In this context first consider Fig. 6.1 b). Here the voltage distribution is extrapolated beyond location $x = 0$ up to the next maximum value. In conjunction with this maximum value there is a positive real resistance $R > Z_0$ where

$$R = \frac{1+\left|\underline{r}\right|}{1-\left|\underline{r}\right|} \cdot Z_0 \tag{6.9}$$

The line segment with the length l, terminated with R, has the same input impedance Z as the original circuit. Thus, for the phase ϕ it is true that

$$\phi = -\frac{4\pi}{\lambda_g} \cdot l = -2\,ß \cdot l \tag{6.10}$$

On the other hand, due to $l + l' = \lambda_g/4$ (distance of the minimum to the maximum) and $\Delta x + l' = \lambda_g/2$ (generally: $\Delta x + l' = n \cdot \lambda_g/2$) the following relationship applies for $\boldsymbol{l}$

$$l = \Delta x - (2n-1)\frac{\lambda_g}{4} \tag{6.11}$$

and therewith

$$\phi/_{\text{rad}} = -4\pi\frac{\Delta x}{\lambda_g} + (2n-1)\pi \quad \text{or}$$

$$\phi = -720°\frac{\Delta x}{\lambda_g} + 180°$$

$$(n = 0, \pm1, \pm2, \ldots) \tag{6.12}$$

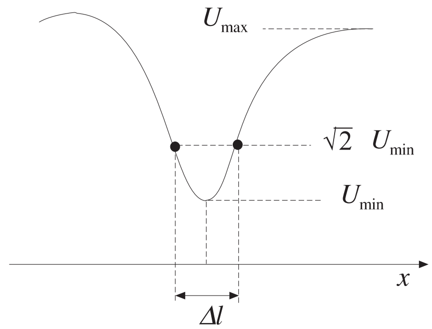

Fig. 6.2: For the determination of s and |r| from the measurement of the "node width" Δl.

To determine ϕ you can proceed as follows:
(a) You determine a location x_0 for the zero point or minimum when the transmission line is shorted.
(b) Afterwards you determine a location x_1 for the minimum when the line is terminated with an unknown impedance.
(c) Δx constitutes the shift from x_1 with respect to x_0. For this Δx is counted as positive, if you must shift from the minimum in the direction of the termination (Z) (see also Fig. 6.1).

The determination of the magnitude $|r|$ out of s, i.e. the ratio of the maximum to the minimum amplitude, becomes inexact as soon as the amplitude ratio (= s) becomes somewhat greater than 5. It is here namely that slight measurement errors arise through the overload of the measuring amplifiers, deviation in the diode characteristic when measuring the maximum and on account of noise during the measurement of the minimum. These errors can be avoided to a great extent, if instead of measuring the voltage quotients $s = U_{max}/U_{min}$ you determine the width of the voltage minimum (node width). If according to Fig. 6.2 you express the so-called node width at $\left|\underline{U}(x)\right|/U_{min} = \sqrt{2}$ (corresponding to 3 dB) with Δl, the exact result is:

$$\frac{1}{s} = \frac{\sin\left(\pi\frac{\Delta l}{\lambda_g}\right)}{\sqrt{1+\sin^2\left(\pi\frac{\Delta l}{\lambda_g}\right)}} \tag{6.13}$$

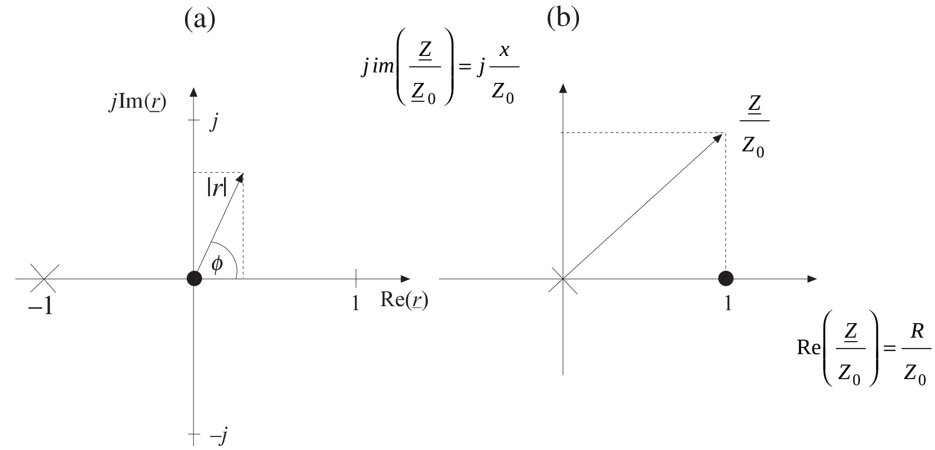

Fig. 6.3: Complex number plane
(a) the reflection coefficient $r = |r| \exp(j\varphi)$ and
(b) for the normed impedance $Z/Z_0 = R/Z_0 + j\,X/Z_0$.

╳: Short-circuit ●: Matching

For $\Delta l/\lambda_g << 1$ this can be approximated with

$$\sin\alpha \approx \alpha << 1 \text{ for } \alpha << 1$$

yielding

$$\frac{1}{s} \approx \pi\frac{\Delta l}{\lambda_g} \tag{6.14}$$

The value $|r|$ is extrapolated from s using Equation (6.8).

Smith chart

The reflection coefficient r and the normalized impedance Z/Z_0 are complex numbers depicted accordingly in separate number planes in Fig. 6.3. If according to Equation (6.3) the corresponding values of Z/Z_0 for the reflection coefficient r are entered into the complex number plane (Fig. 6.3 a) and this is done in the form of lines of the constant real component R/Z_0 and constant imaginary component X/Z_0, you obtain the *Smith Chart* depicted in Fig. 6.4. It is well suited for the rapid determination of Z/Z_0 for a given value of r (and vice versa). Furthermore, it is very useful – as explained in more detail in Experiment 7– for describing various properties of transmission line networks. In Fig. 6.4 the reflection coefficient is entered for $Z/Z_0 = 0.5 + j\,1.4$. By dimensioning the length $|r|$ relative to the radius of the outer circle you obtain $|r| \approx 0.72$ and by measuring the angle you arrive at $\phi \approx 67°$. The determination of Z/Z_0 at a given value of r is performed in a similar manner.

The Smith chart can also be used for admittances $Y\,Z_0 = Z_0/Z$. The corresponding complex number is then ($-r$). Then if it is true that $Y/Y_0 = Y\,Z_0 = 0.5 + j\,1.4$, the corresponding reflection coefficient is:
$0.72 \cdot \exp[j\,(67°–180°)] = 0.72 \cdot \exp(-j\,113°)$.

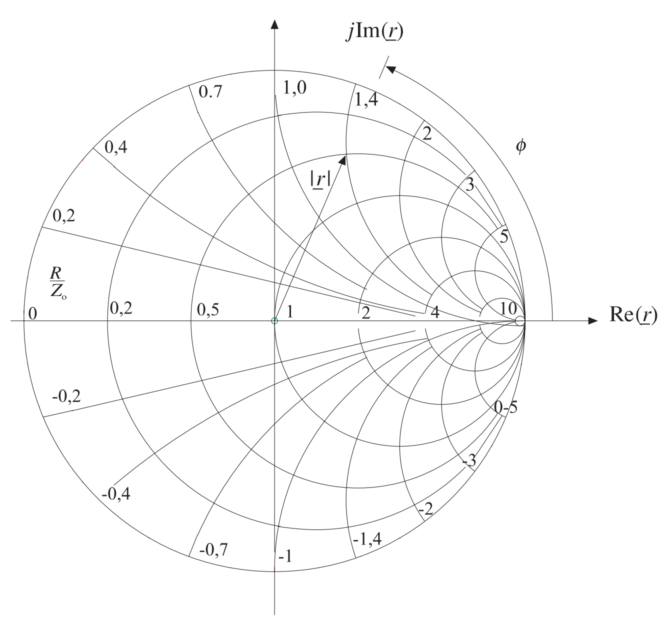

Fig. 6.4 Smith chart

Note:
As is shown in Experiment 3 of MTS 7.4.5, "any" random reflection coefficient (load) can be realized using the combination (series connection) of a variable attenuator, a variable phase-shifter and a short-circuit plate. Here the attenuator primarily influences the magnitude and the phase-shifter affects the phase-angle of the reflection coefficient.

Required equipment

1 Basic unit	737 021
1 Gunn oscillator	737 01
1 Diaphragm with slit 2 x 15 mm 90°	737 22
1 Slotted measuring line	737 111
1 Coax detector	737 03
1 Short-circuit plate, from accessories	737 29
1 Sample holder, from accessories	737 29
1 Sample made of graphite(*), from accessories	737 29
1 Waveguide termination	737 14
1 Set of thumb screws (2 each)	737 399

Additionally required equipment

1 Oscilloscope (optional)	575 29
1 XY recorder (optional)	575 663
2 Coaxial cables with BNC/BNC plugs, 2 m	501 022
2 Stand bases	301 21
2 Supports f. waveguide components	737 15
1 Stand rod 0.25 m	301 26

Recommended

1 PIN modulator	737 05
1 Isolator	737 06

Experiment procedure

1. *Set up the experiment configuration as specified in Fig. 6.5.*
   Note:
   The diaphragm with slit again serves as a filter to improve spectral purity.

2. *Calibration*
   Screw on the short-circuit plate to the end the slotted measuring line to function as the device under test (DUT). Determine the location x_0/mm of the first minimum (zero point) in front of the short-circuit plate and enter the findings in Table 6.1.

3. *Measurement of the sample*

3.1 Assemble a measurement object A consisting of a sample holder with graphite material sample (see Fig. 6.5) and a reflection-free waveguide termination. This measurement object is fastened onto the open end of the slotted measuring line.

3.2 Determine the standing wave ratio (VSWR) s with the help of the scale on the frequency-selective voltmeter. To do this, calibrate in the voltage maximum to "0 dB" using the range switch and the

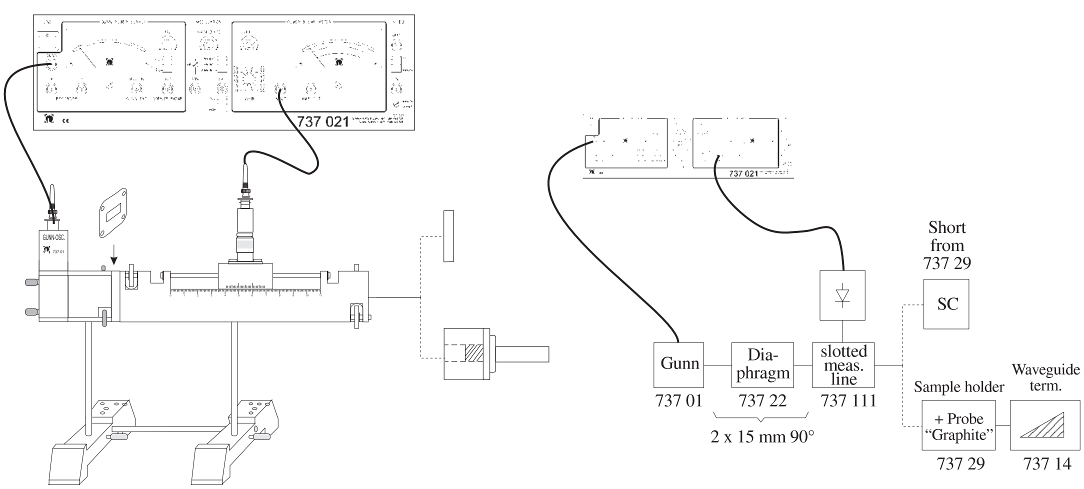

Fig. 6.5 Experiment setup

(*) : In order to be able to compare the measured values of this experiment with those from Experiment 11, the samples used here should be stored so as not to mix them up with any other existing samples.

"ZERO" controller. Shift the position of the slotted measuring line probe to the first minimum in front of the load. This is where you read off the VSWR value and enter it into Table 6.1.

3.3 Determine the location x_1 of the minimum and enter this value into Table 6.1

3.4 Preliminary evaluation:
Determination of $|r|$ from s with the aid of Equation (6.8) and extrapolation of ϕ from $\Delta x = \pm(x_1 - x_0)$ and Equation (6.12). Enter the values into Table 6.1.

3.5 Determining $|r|$ by means of the node width Δl. For this proceed with measurement techniques corresponding to Fig. 6.2 to determine the node width (3 dB beyond U_{min}). Calculate s according to Equation (6.13) and $|r|$ according to Equation (6.8). Enter the findings into Table 6.2
Tip:
Adjust the value of the minimum to –3 dB using the gain selection switch and the "ZERO" controller, and find the positions (to the left and right of the minimum), where the display assumes a value of 0 dB.

Questions

1. Determine the value of the normalized impedance Z/Z_0 for the measurement object A (see Table 6.1) using the Smith chart (approximately with Fig. 6.4 or more accurately with Fig. 7.7 of the subsequent experiment).

Table 6.1

DUT	Location of the Minimum/mm	s	$\|r\|$	ϕ/degree
Short-circuit plate		∞	1	180
Measurement object A				

Table 6.2

	Δl/mm	s	$\|r\|$
Measurement object A			

Bibliography

[1] S.J. Algeri, W.S. Chiung, L.A. Stark: Microwaves Made Simple -The Workbook. Artech House, Norwood 1986

[2] G.J. Wheeler: Introduction to Microwaves. Prentice Hall, New Jersey

[3] T. Moreno: Microwave Transmission Design Data. Dover Publ., New York

[4] C.G. Montgomery: Technique of Microwave Measurements. McGraw Hill

[5] S. F. Adam: Microwave Theory and Applications. Prentice - Hall, Englewood Cliffs, 1969

[6] T. S. Laverghetta: Modern Microwave Measurements and Techniques. Artech House, Norwood 1988

[7] T. A Johnk: Engineering Electromagnetic Fields and Waves. Wiley & Sons, New York 1975