The slotted measuring line

Sampling the field in front of a short-circuit plate and a waveguide termination

Principles

Various line types for electromagnetic waves and their technical function

Lines fulfill various functions in radio frequency technology:

(a) They serve in the transmission of radio frequency signals between remote locations. Examples: radio link between the transmitter and a remote antenna system, broadband cable for signal distribution of a satellite receiver system to the individual subscribers and underwater cables. Transmission lines and directional radio links in free space are frequently considered potential alternatives in this function as a medium for communications transmission

(b) They also serve as circuit elements (instead of capacitors and inductors) and interconnecting lines for the realization of passive microwave circuits (e.g. transmission-line filters).

In general, transmission lines can be considered as structures for guiding electromagnetic waves. For this function a multitude of various transmission line types are suitable.

The coaxial and two-wire line depicted in Fig. 5.1a) guides transverse electromagnetic waves (TEM waves) and can also be used for any arbitrarily low frequency.

In other transmission line types, however, the wave propagation is dependent on the condition that the cross-sectional dimensions of the line are at least about as large as a half free-space wavelength ($\lambda_0/2$). This leads to the result that in the range of frequencies above approx. 1 GHz there are a larger number of transmission line types available than for low frequencies. This is because it is only at the higher frequencies where the dimension $\lambda_0/2$ results in "handy" cross-sections.

Figure 5.1 b) shows various planar transmission line types, which are particularly well-suited for the design of microwave integrated circuits (MIC).

Transmission lines can be realized without any metal conductors, see Fig. 5.1 d). The dielectric waveguides shown there are based on the principle of total reflection of electromagnetic waves at the boundary surfaces between the insulators with "higher" to "lower" relative permittivity ε_r.

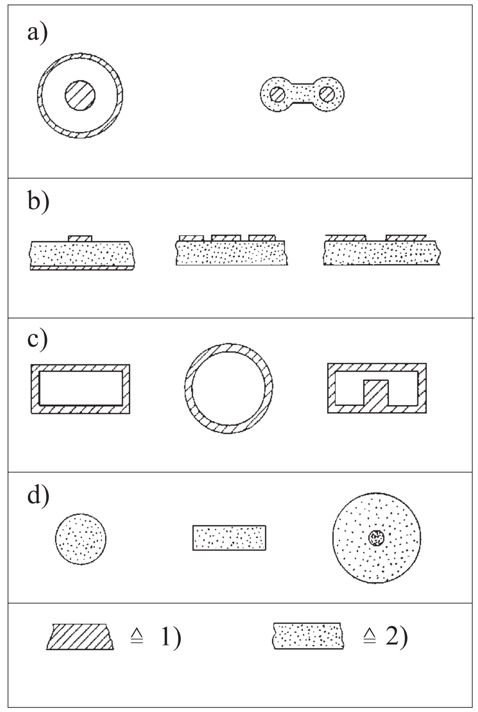

Fig. 5.1: Various line types for electromagnetic waves (1 = metal conductor, 2 = isolator). Numbering from left to right.
(a) Coaxial and two-wire lines
(b) Planar line structures: Microstrip line, coplanar line, slotted line
(c) Waveguide: Rectangular, circular and ridged waveguides
(d) Dielectric waveguides: Round and rectangular-shaped surface waveguide; optical waveguide

Necessary fundamentals drawn from elemtary transmission line theory

For the description of line-bound wave propagation we can begin our investigation with a

simple two-wire line on which the voltage and current can be specified at any given location. Subsequently the results can be generally applied as a model for any other homogenous transmission line.
Fig. 5.2 shows a homogenous transmission line structure represented symbolically by two double lines. The electromagnetic state on this line can be expressed by specifying the spatial and time dependency of the instantaneous voltage *u(x,t)* and current *i (x,t)*.
First we will consider the case in which a single wave propagates on the line in the +x direction. The following holds true with $\omega = 2\pi f$ as the angular frequency and λ_g as the guided wavelength ("*g*" = guide) and where the attenuation on the line is disregarded

$$u(x,t) = \hat{u} \cdot \cos\left(\omega \cdot t - 2\pi \frac{x}{\lambda_g}\right) \qquad (5.1)$$

The phase velocity resulting here is

$$v_{ph} = \frac{\omega \cdot \lambda_g}{2\pi} = f \cdot \lambda_g \qquad (5.2)$$

If Z_0 expresses the line's characteristic impedance, then the following holds true for the corresponding voltage *u (x,t)* and current *i (x, t)*

$$i(x,t) = \frac{u(x,t)}{Z_0} \qquad (5.3)$$

In the description of time-harmonic processes you can also make use of complex amplitudes. As such an AC voltage of the form $u(t) = \hat{u} \cos(\omega t + \phi)$ can be expressed by a corresponding complex amplitude $U = \hat{u} \exp(j\phi)$ and the following applies

$$u(t) = \mathrm{Re}\{U \cdot \exp(j\,\omega\, t)\} \qquad (5.4)$$

Complex numbers are now specially marked by an underline. Equations (5.1) and (5.3) are given complex numbers with the form

$$\underline{U}(x) = |\underline{U}(x)| \cdot \exp[j\phi_u(x)] \qquad (5.5)$$

where $|\underline{U}(x)| = \hat{u}$

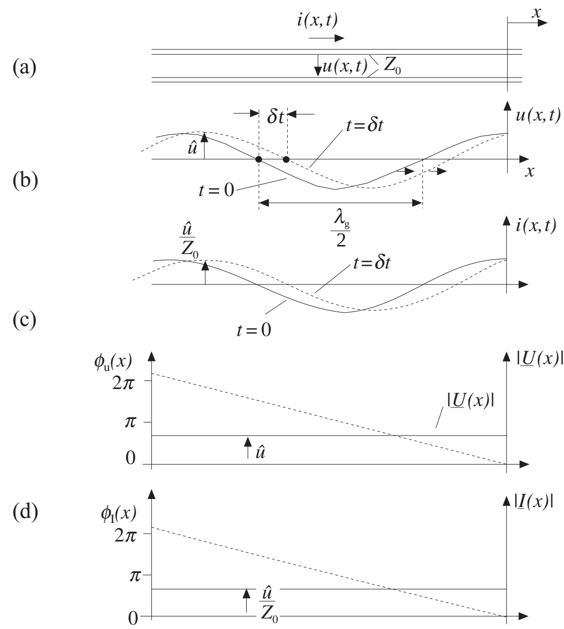

Fig. 5.2: Wave propagation on a line
(a) Spatial dependency of the voltage *u (x,t)* at two different time points
(b) Spatial dependency of the current *i (x,t)* at two different time points
(c) Spatial dependency of the magnitude |*U (x)*| and phase Φ_u *(x)* for the complex voltage amplitude (phasor)
(d) Spatial dependency of the magnitude |*I(x)*| and phase Φ_I *(x)* for the complex current amplitude (phasor).

and $$\phi_u(x) = -2\pi \frac{x}{\lambda_g} = -\,ß \cdot x \qquad (5.6)$$

and as such the following is true for the current

$$\underline{I}(x) = |\underline{I}(x)| \cdot \exp[j\phi_I(x)] \qquad (5.7)$$

where

$$|\underline{I}(x)| = \frac{\hat{u}}{Z_0} \text{ und } \phi_I(x) = \phi_u(x) = -\,ß \cdot x \qquad (5.8)$$

Figures 5.2 c) and d) show the spatial dependency of the magnitudes |*U (x)*| and |*I (x)*| as well as the phases Φ_u *(x)* and Φ_I *(x)* of the complex voltage and current amplitude. In equations (5.6) and (5.8) the following phase constant has been introduced

$$ß = \frac{2\pi}{\lambda_g} = \frac{\omega}{v_{ph}} \qquad (5.9)$$

as a second parameter to express line-bound wave propagation.

Description of the field in front of a short-circuit

If there is a short at the end of the transmission line ($x = 0$) (see Fig. 5.3, above), the wave can be described from the superposition of one component travelling to the load $u_+(x, t)$ and one reflected component travelling back from the load $u\_(x, t)$.

As the total voltage at $x = 0$ must be identical to zero (short-circuit!), it follows that:

$$\begin{aligned} u(x, t) &= u_+(x, t) + u\_(x, t) \\ &= \hat{u} \cos(\omega t - \beta x) - \hat{u} \cos(\omega t + \beta x) \\ &= 2 \cdot \hat{u} \cdot \sin(\omega t) \cdot \sin(\beta x) \end{aligned} \qquad (5.10)$$

The current of the reflected wave is related to $u\_$ by virtue of ($-Z_0$) and thus it follows (see Fig. 5.3 b) that

$$i(x,t) = \frac{2 \cdot \hat{u}}{Z_0} \cdot \cos(\omega \cdot t) \cdot \cos(\beta \cdot x) \qquad (5.11)$$

The current and voltage now have distinct responses with respect to time and space. We are dealing with a standing wave [see Fig. 5.3 c) and d)]. The nodes and maxima of the u and i characteristic are at locations unchanged with respect to time. The nodes of the voltage are shifted by $\lambda_g/4$ with respect to the nodes of the current. Moreover, there is a phase shift of $+\pi/2$ between the voltage and current. This means that u exhibits its extreme values exactly when i (for each x) is zero and vice versa. If these relationships are represented for the magnitude and the phase of the complex amplitude, then we obtain the results depicted in Figures 5.3 e) and f).

Wave propagation in a rectangular waveguide

Generally a waveguide is a "hollow tube" in which electromagnetic waves can propagate. A rectangular waveguide as shown in Figure 5.4 is a special form of waveguide. This rectangular waveguide is the object of a series of experiments and should therefore be considered more closely in theoretical terms. This is also because many of the results found for rectangular waveguides also apply to other forms of waveguides (e.g. circular waveguide).

If you consider the zig-zag reflection of a plane uniform wave between the side surfaces (distance **a**) of a waveguide, you obtain a critical frequency called the cut-off frequency, as of which wave propagation becomes possible

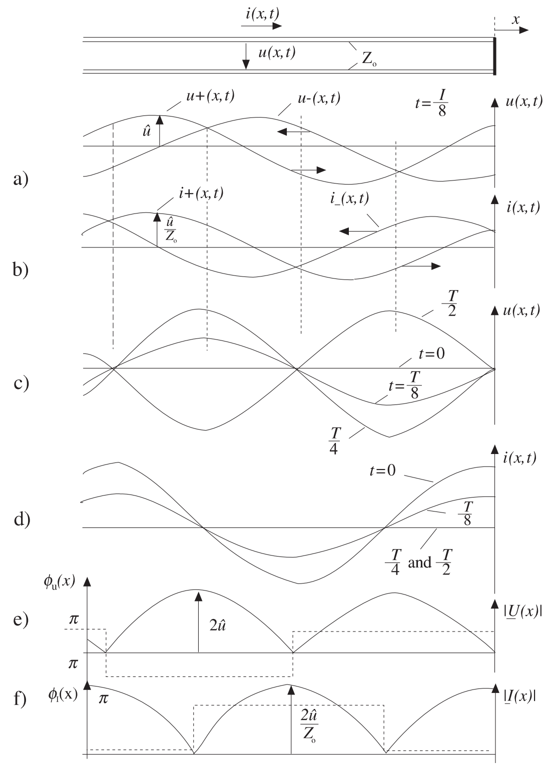

Fig. 5.3: Standing wave on a line shorted on one end:
- a) Voltage characteristic $u_+(x, t)$ of the wave travelling to the load and $u\_(x, t)$ of the reflected wave travelling back at the time point $t = T/8$.
- b) $i_+(x,t)$ and $i\_(x,t)$ at $t = T/8$.
- c) Characteristic of the total voltage $u(x, t) = u_+(x, t) + u\_(x, t)$ at 4 different points in time
- d) $i(x, t) = i_+(x, t) + i\_(x, t)$
- e) Spatial dependency of the magnitude $|\underline{U}(x)|$ and phase $\Phi_u(x)$ of the complex voltage amplitude
- f) Spatial dependency of the magnitude $|\underline{I}(x)|$ and phase $\Phi_I(x)$ of the complex current amplitude.

$$f_c = \frac{c}{\lambda_{c,0}} = \frac{c}{2a} \qquad (5.12)$$

For the phase velocity a relationship is yielded

$$\upsilon_{ph} = \frac{c}{\sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2}} \qquad (5.13)$$

A calculation of the guided wavelength λ_g and the phase constant $ß$ amounts to

$$\lambda_g = \frac{v_{ph}}{f} = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2}} \qquad (5.14)$$

$$ß = \frac{2\pi}{\lambda_g} = \frac{2\pi}{\lambda_0}\sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2} \qquad (5.15)$$

For more information regarding the derivation of these relationships we wish to refer to the specified sources in the bibliography.
Another approach used for describing propagation originates from the solution of Maxwell's equations with the boundary conditions existing at the metal surfaces taken into account. But do not be alarmed. Here only the result of this analysis is reproduced for the fundamental mode (TE_{10}):
The electrical field has only one Cartesian component. It points in the y-direction and so it follows that:

$$\underline{E}_y(x,z) = \sqrt{\frac{2}{a \cdot b}} \cdot \underline{U}_0 \cdot \sin\left(\frac{\pi}{a}x\right) \cdot e^{-jßz} \qquad (5.16)$$

It is evident that E_y is not only dependent on the propagation coordinate z, but also on the transverse coordinate x. As the tangential components of the electrical field must vanish at the surface of (ideal) conductors, it is true that

$$E_y\,(x = 0,\ z) = E_y\,(x = a,\ z) = 0.$$

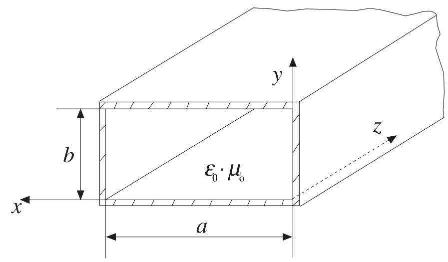

Fig. 5.4: Rectangular waveguide

The field strength is maximum in the middle of the waveguide at $x = a/2$, and it holds true that

$$\left|\underline{E}_{y,\,max}\right| = \sqrt{\frac{2}{a \cdot b}} \cdot \left|\underline{U}_0\right|$$

whereby U_0 constitutes an arbitrarily introduced voltage.

Its corresponding magnetic field has both a transverse component $H_x(x, z)$ as well as a longitudinal component $H_z(x, t)$.
The transverse component H_x has the same phase as E_y and also the same spatial dependency, and so it follows that:

$$\underline{H}_x(x,z) = -\frac{ß}{\omega\mu_0} \cdot \sqrt{\frac{2}{ab}} \cdot \underline{U}_0 \cdot \sin\left(\frac{\pi}{a}x\right) e^{-jßz} \qquad (5.17)$$

At every cross-sectional point z = const. the ratio of E_y to $(-H_x)$ is equal and given by the characteristic impedance

$$Z_0 = \frac{\underline{E}_y(x,z)}{\left(-\underline{H}_x(x,z)\right)} = \frac{\omega\mu_0}{ß} = \frac{120 \cdot \pi}{\sqrt{1-\left(\frac{\lambda_0}{2 \cdot a}\right)^2}} \qquad (5.18)$$

The following applies for the longitudinal component H_z:

$$\underline{H}_z(x,z) = j\frac{\pi}{a}\frac{1}{\omega\mu_0}\sqrt{\frac{2}{ab}} \cdot \underline{U}_0 \cos\left(\frac{\pi}{a}x\right) \cdot e^{-jßz} \qquad (5.19)$$

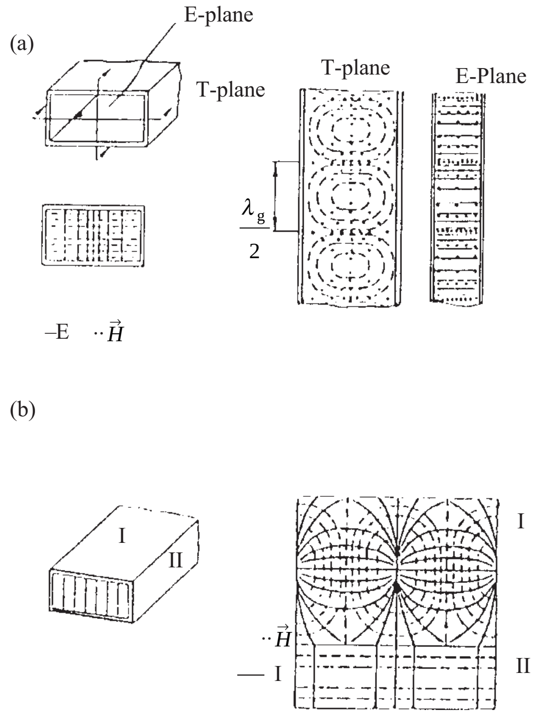

Fig. 5.5: Field pattern of the fundamental mode (TE_{10}-wave) in the rectangular waveguide
(a) E and T fields
(b) Current density distribution on the metal walls

Thus it is phase-shifted by $\pi/2$ with respect to the transverse components of $\vec{E}$ and $\vec{H}$. In contrast to these it assumes at $x = 0$ and $x = a$ its maximum value (in terms of magnitude) at the metal sidewalls. The spatial dependencies of the field components reproduced in Equations (5.16) to (5.19) are shown in Fig. 5.5. The wave type under consideration till now is also referred to as "TE_{10}-mode". Here the TE stands for transverse electric and "1" means that the number of halfwaves in the x-direction is 1 while the index 0 means that the field is constant in the y-direction. At higher frequencies higher wave modes, namely TE_{mn}-waves and TM_{mn}-waves are capable of propagation.
According to Fig. 5.6 the TE_{20} mode is capable of propagation starting from a frequency of $f_{c2} = 2\,f_{c,TE10}$, when there is a side ratio of $a/b = 2.25$ so that a "frequency octave" is available for single-mode operation.

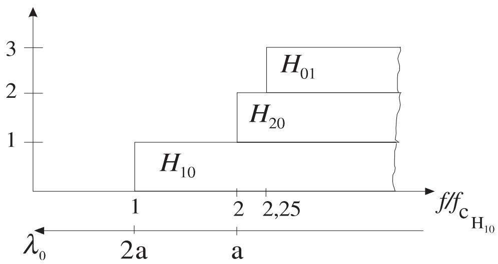

Fig. 5.6: Existence zones for various wavetypes capable of propagating in the rectangular waveguide ($a = 2.25\ b$).

Slotted measuring line

As will be demonstrated with particular care in experiment 6, a reflection at the end of the transmission line has the effect that maxima and minima are formed in the spatial distribution of the field strength along the line. Based on the ratio of the amplitude values (maximum/minimum) and the locations of the maxima and minima you can draw conclusions as to the magnitude and phase of the reflection coefficient.
If to this purpose you wish to measure the distribution of the field strength along the transmission line (slotted line), the following must be taken into consideration:

(a) You should interfere as little as possible with the electromagnetic field in the waveguide. This requirement is met, if a "narrow" slit is added in the center of the wide section of the waveguide (see Fig. 5.7 and compare to Fig. 5.5 below).

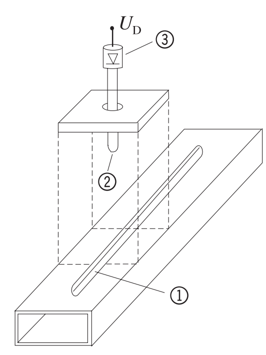

Fig. 5.7: Principle of the slotted measuring line (exploded view)
① Slit in the waveguide
② Field probe = short rod-type antenna
③ Detector diode

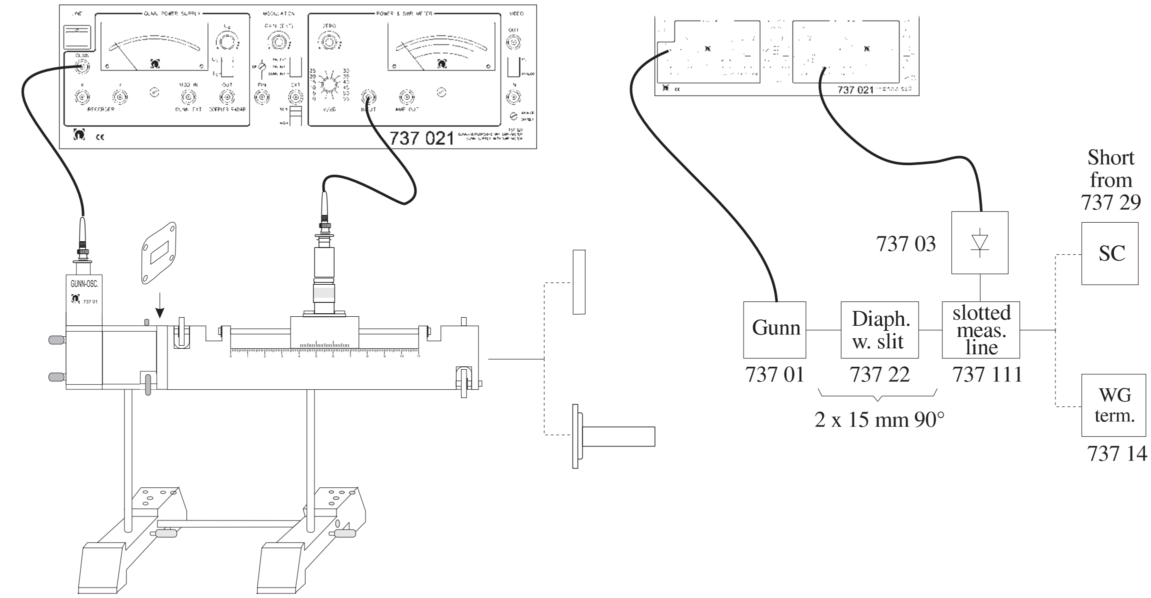

Fig. 5.8: Experiment setup

(b) The characteristic impedance Z_0 (see Equation [5.19]) is slightly altered by this modification. This can be compensated for by slightly increasing the waveguide width a in the region of the slit.

(c) A short probe ("electrical dipole") as shown in Fig. 5.7 supplies a voltage to its output which is proportional to the transverse component $|E_y|$ of the electrical field strength. Thus behind the detector probe (square-law rectification) you obtain a voltage

$$U_D = K \cdot \left|\underline{E}_y\right|^2 \qquad (5.20)$$

Here K is a constant with the dimension $\frac{m^2}{V}$

Low reflection waveguide termination

By inserting a wedge-shaped absorber material the power of the incident wave is nearly completely absorbed thus suppressing any reflection almost totally.

Required equipment

1 Basic unit	737 021
1 Gunn oscillator	737 01
1 Diaphragm with slit 2 x 15 mm 90°	737 22
1 Slotted measuring line	737 111
1 Coax detector	737 03
1 Short-circuit plate, from accessories	737 29
1 Waveguide termination	737 14
1 Set of thumb screws (2 each)	737 399

Additionally required equipment

1 Oscilloscope (optional)	575 29
1 XY recorder (optional)	575 663
2 Stand bases	301 21
2 Supports for waveguide components	737 15
1 Stand rod 0.25 m	301 26
2 Coaxial cables with BNC/BNC plugs, 2 m	501 022

Recommended

1 PIN modulator	737 05
1 Isolator	737 06

Experiment procedure

Note:

If you are using a PIN modulator and isolator the experiment setup in Fig. 5.8 is supplemented as explained in the preface.

1. *Set up the experiment arrangement in accordance with Fig. 5.8 (perhaps modify according to Fig. 0.5)*

   Note:

   The diaphragm slit serves as a frequency-selective component (filter) to improve the spectral purity of the guided wave.

2. *Measurement with short-circuit plate*

2.1 Attach the short-circuit plate at the open end of the slotted measuring line.

2.2 Set the range switch V/dB of the SWR meter to the most insensitive range.
Set the Gunn voltage to between 8 V and 9 V (Tip: first increase it to 10 V and then adjust it back to the desired value), set the modulation switch to GUNN-INT.

2.3 Place the slotted measuring line probe at one end position. Now slowly push the slide to the other end position and at the same time search for the maximum detector voltage (always adjusting the gain to a suitable level). Calibrate the maximum value to “0” dB using the “ZERO” controller. The maximum position is designated x_0 (read off the scale and note down) and functions as the reference position.

2.4 Now shift the position of the probe in 2 mm steps (in the direction of the end position which is farthest removed from x_0), in other words to the corresponding positions $|x_n - x_0| = n \cdot 2$ mm. Enter the values measured for the detector voltage into the second column of Table 5.1.

Table 5.1

Probe position $\lvert x - x_0 \rvert$ in mm	Display in dB $= 20 \cdot \log \frac{U(\lvert x - x_0 \rvert)}{U_{max}}$	$\frac{U(\lvert x - x_0 \rvert)}{U_{max}}$	$\left\lvert \cos\left(\frac{2\pi}{\lambda_g} \cdot \lvert x - x_0 \rvert\right) \right\rvert$
0			
2			
4			
6			
8			
10			
12			
14			
16			
18			
20			
22			
24			
26			
28			
Minimum at			
Distance of the Minima Δx / mm=_________	λ_g =_________ mm		

Table 5.2

Probe position $\lvert x - x_0 \rvert$ in mm	Display in dB $= 20 \cdot \log \frac{U(\lvert x - x_0 \rvert)}{U_{max}}$	$\frac{U(\lvert x - x_0 \rvert)}{U_{max}}$
0		
2		
4		
6		
8		
10		
12		
14		
16		
18		
20		
22		
24		
26		
28		

You can conclude the measurement after exceeding the position of the 2nd maximum (Display/dB ≈ 0).

2.5 Determine the distance Δx/mm between two minima and enter the result in the last line of Table 5.1. (Tip: By successively increasing the gain (V/dB) in the minima, you can determine the positions more precisely).

3. *Measurement with reflection-free waveguide termination.*

3.1 The short-circuit plate is replaced by the “reflection-free” waveguide termination.

3.2 Slide the probe along the slotted measuring line over the entire range and note down by which value (in dB) there is deviation in the characteristic, i.e. by which value the level (Δa) has dropped in comparison to the short-circuit experiment.

3.3 As in point 2.3 move the probe over the entire range and search for the location of the maximum detector voltage and again re-calibrate this value to 0 dB.

3.4 Position the probe in 2-mm steps in the same locations as in Experiment 2.4 (see also Table 5.1), and enter the display values in Table 5.2.

Note:

If you have an XY recorder, a digital storage oscilloscope or a CASSY interface at your disposal you can also directly record the measurement curve. For this purpose the slotted measuring line is equipped with an integrated displacement sensor.

- Connect the IN socket of the slotted measuring line to the X output (supplying $|U_G|$) of the basic unit (or with an external power supply with 10 V DC).
- Connect the X socket of the slotted measuring line to the X input of the XY recorder (oscilloscope or CASSY).
- Connect the Y input of the XY recorder (oscilloscope or CASSY) to the AMP-OUT socket of the basic unit.

The AMP-OUT socket supplies a linear output signal whereby 0 V is about –20 dB and approx. 4.5 V corresponds to 0 dB.

Furthermore, it is important to point out that the integrated displacement sensor is not completely backlash-free (it does possess hysteresis), for that reason the curve should only be plotted once in one direction.

Questions

1. Based on the distance between two neighboring zero points (distance to the minima) Δx/mm (see Table 5.1) determine the wavelength λ_g of the guided wave in mm.

2. Using the value for λ_g determined above under point 1 with the waveguide width of $a = 22.9$ mm taken into consideration, determine the free-space wavelength λ_0 and frequency f of the guided wave.

3. What is the mathematical value resulting for the phase velocity v_{ph} and the phase constant β according to the Equations (5.13) and (5.15).

4. What is the cut-off frequency f_c of the TE_{10} mode in the given rectangular waveguide, and from which frequency is the TE_{20} wave capable of propagation? For this also refer to Fig. 5.6.

5. In addition to the values from the measuring instrument displays determined through experimentation in experiment 2.4 calculate the ratio of the respective voltages (field strength levels) to the maximum value and enter these in column 3 of Table 5.1.
   Note:
   As described in Experiment 2 the detector voltage is proportional to the square of the received field strength (or here the voltage from the transmission line model according to Fig. 5.2). You take the logarithm of the value indicated in the SWR meter display in accordance with the expression $10 \cdot \log(U_D \,/\, U_{D,ref})$. From this you obtain the relationship

$$\frac{\text{Display}}{\text{dB}} = 20 \cdot \log \frac{U\left(\left|x - x_0\right|\right)}{U_{max}}$$

   where $U_{max} = \left|U\left(x_0\right)\right|$

6. According to Fig. 5.3 e) the voltage (field strength) responds accordingly.

$$\frac{U\left(\left|x - x_0\right|\right)}{U_{max}} = \cos\left(\frac{2\pi}{\lambda_g} \cdot \left|x - x_0\right|\right)$$

   [*Note:*
   In Equation 5.10 the sinusoidal term is defined as $\sin(\beta x)$. However, here it is permitted to convert to the cosine term because it only concerns a phase-shift and a maximum is assumed to exist at $x - x_0 = 0$].
   The values resulting from this equation are entered into column 4 of Table 5.1 and then they are compared to the values in column 3. Discuss any possible causes if you notice regular deviations between the values in the two last columns.

7. In conjunction with the values determined experimentally under point 3.4 from the measuring instrument display, calculate the ratio of the voltage (field strength) to the voltage maximum at the respective location of the slotted measuring line. Enter the values into the 3rd column of Table 5.2.

8. Discuss the value Δ obtained under experiment point 3.2.
   By what magnitude in dB do the measured values from subpoint 3.4 vary?